A team-based deployment approach for heterogeneous mobile sensor networks

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Abstract

This paper presents a distributed algorithm for deploying teams of heterogeneous agents to cover multiple regions of interest. A team-based approach is proposed here to minimize a locational cost function, defined with respect to various regions of interest, while each region is covered by intended agents. The main region is first partitioned into smaller regions among teams using the so-called Power diagram in such a way that larger regions are assigned to those teams that have higher capabilities. The immediate consequence of the difference between heterogeneous teams is an additional term that appears in control laws of their corresponding agents, which is determined by some calculations along their boundaries. The teams’ assigned regions are then partitioned among their members by the so-called multiplicatively-weighted (MW) Voronoi diagrams with guaranteed collision avoidance. A distributed control law is developed based on partitioning in team and agent levels to guarantee the convergence of agents to locally optimal positions. Numerical results are presented to illustrate the effectiveness of the proposed team-based weighted partitioning methods that enable distributed deployment of teams of heterogeneous agents.

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1. Introduction

Deployment of a group of agents to perform a distributed task has found various applications in environmental monitoring, sensing, surveillance, search and data collection, and map building, among many others (Bullo, Cortés, & Martinez, 2009). The deployment problem is formulated as a locational optimization problem to cover a region of interest in the environment. The known environment is then divided into regions, assigned to individual agents, in order to minimize an appropriately defined cost function. Subsequently, distributed control laws determine positions of agents within their own regions.

Gradient descent-based coverage control laws, using the Voronoi-based locational optimization framework, have been proposed for navigating agents to reach the centroids of Voronoi cells, calculated by distributed partitioning of a known environment (Cortés, Martinez, Karatas, & Bullo, 2004). This framework has been extended to address more practical constraints such as non-convex environments, obstacle and collision avoidance, time-varying environment, and limited communication ranges (Abbasi, Mesbahi, & Velni, 2019; Kantaros, Thanou, & Tzes, 2015; Kantaros & Zavlanos, 2016; Miah, Panah, Fallah, & Spinello, 2017; Nowzari & Cortés, 2012). However, it has been assumed that sensing and communication capabilities of all agents are identical.

A fundamental challenge in deploying a heterogeneous group of agents to perform a distributed task is the fact that agents vary in the quality of performing assigned tasks. The performance of each agent depends on its physical capabilities such as sensor setup, maximum traveling speed and size (Dou, Song, Wang, Liu, & Feng, 2017; Mahboubi, Moezzi, Aghdam, & Sayrafian-Pour, 2017; Song, Liu, Feng, & Xu, 2016). More realistic applications have been addressed by the use of generalized Voronoi diagrams, since they offer a higher degree of freedom in assigning regions to agents. For instance, weights associated with generalized Voronoi partitioning can modify regions assigned to each agent without the need to change the location of the agent. A Power diagram, which has strong similarities to Voronoi diagrams, has been applied in Pavone, Frazzoli, and Bullo (2011) to provide an equitable workload sharing to coverage problems. Additionally, variations in sensing performance have been taken into account by adjusting weights of Power diagrams (Pierson, Figueiredo, Pimenta, & Schwager, 2015; Pierson & Schwager, 2016). An extended Power diagram has been proposed in Kantaros et al. (2015) in order to address differences in visibility and sensing capabilities of...
agents. Agents with different sensing capabilities and operating costs have been deployed to solve the coverage problem by using multiplicatively-weighted Voronoi (MW-Voronoi) diagrams (Mahboubi & Aghdam, 2017; Sharifi, Chamseeddeh, Mahboubi, Zhang, & Aghdam, 2015). The weights of an MW-Voronoi diagram are adjusted according to the sensing radius to discover undetectable regions in an environment (Mahboubi & Aghdam, 2017).

In practical applications, an environment may contain several regions of interest (Abbasi, Mesbahi, & Velni, 2017a; Abbasi, Mesbahi, Velni, & Li, 2018; Nunes, McIntire, & Gini, 2016). By taking into account the differences in the capabilities of agents, specific types of agents may be expected to be assigned to each region of interest (Grocholsky, Keller, Kumar, & Pappas, 2006; Murphy et al., 1999; Wurm, Dornhege, Nebel, Burgard, & Stachniss, 2013). As an example, in performing search and rescue missions, not only are an aerial agent and a large agent needed to cover a region of interest, but smaller agents are also needed in order to reach otherwise inaccessible regions. A team-based framework has been proposed in order to guarantee that each region of interest is covered by the intended agents (Abbasi, Mesbahi, & Velni, 2016, 2017b). However, the heterogeneity of agents has been ignored in assigning regions to teams and their agents.

In the present work, generalized Voronoi diagrams are utilized to take heterogeneous properties and capabilities of agents into account by using the concept of a team, in which each agent can team up with other agents based on the regions of interest and available resources. Then, the environment is partitioned among the teams including heterogeneous members such that larger subregions are assigned to teams with higher capabilities. A weight factor is assigned to each team according to the capabilities of its member agents. In the proposed coverage formulation, the region of interest and weight factor associated with each team differ from its neighboring teams. Therefore, unlike existing coverage algorithms, the proposed distributed control laws include an additional term to capture the potential differences in the region of interest and weight factor of the team with its neighboring teams, and that additional term is determined through some algebraic calculations along the boundaries of neighboring agents. Those weights are used to represent heterogeneity of the deployed agents. It was shown in Pavone et al. (2011) that the difference in the agents weights is one way to capture the varying sensing responsibility (or capability) by modifying the area assigned to each agent (Pavone, Arsie, Frazzoli, & Bullo, 2009). Hence, a smaller relative weight would result in a smaller assigned coverage area for an agent. Another way to look at these weights is to see them as energy-efficiency metrics. This immediately implies that agents with smaller weights tend to cover smaller areas due to their lower efficiency factors.

Among the generalized Voronoi diagrams, the boundaries between two neighbor’s Power cells are generally straight lines (Okabe, Boots, & Sugihara, 1992). Therefore, Power diagrams are the most suitable generalized Voronoi diagram for team-level partitioning, because calculations along their boundaries, which are straight lines, are easier to manage than boundaries of other generalized Voronoi diagrams. Another advantage of using the Power diagrams is the fact that they result in a convex partitioning of the main region (Aurenhammer, 1987). In the next step, each team’s assigned subregion is divided among its members by solving another partitioning problem at the agents level. Although the Power diagram is an appropriate choice for team-level partitioning, it is not suitable for agent-level partitioning because of its negative impacts on coverage performance. First issue is that a member agent might fall outside its assigned cell (Okabe et al., 1992). In that case, each agent would need to be moved towards a point inside its cell in order to obtain a locally optimal solution. Therefore, there is a possibility of collision of agents located outside their cells with other members. Second, Power cells calculated for some of the agents might be empty with respect to the initial location of members (Okabe et al., 1992). Therefore, those members do not move and hence play no role in improving performance. However, the abandoned member could have improved the local coverage solution if it had been located in a different initial position. Based on the above discussion, Power diagrams are the best option for team-level partitioning because of straight line boundaries and convex partitions, but are not the best choice for agent-level partitioning because of the possibility of collisions, a slow convergence rate and abandoned members. All of the mentioned disadvantages in the agent-level partitioning can be avoided by using the MW-Voronoi diagrams. The boundaries of the MW-Voronoi diagrams are easily calculated by computing Apollonian circles (Okabe et al., 1992). Since the importance functions associated with the member agents in each team are the same, there is no need for any calculations along the boundary of neighboring member agents in agent-level partitioning. Therefore, we propose to use the MW-Voronoi diagrams to perform the agent-level partitioning, which also guarantees collision avoidance due to the properties of the MW-Voronoi diagrams.

The remainder of this paper is structured as follows. Definitions and the problem statement are provided in Section 2. The two-step optimization problem with respect to the Power diagram and the MW-Voronoi diagram is also described in Section 2. Section 3 introduces a new strategy suited for the coverage control in the presence of differences in the importance functions and service factors associated with the neighboring teams. Then, two different approaches are suggested for partitioning the subregion assigned to each team among its member agents. Section 4 presents simulation results to illustrate the proposed team-based solution method for the coverage problem, and finally Section 5 concludes the paper.

Notations 1. We use \( \mathbb{N}, \mathbb{R} \) and \( \mathbb{R}_+ \) to denote the sets of natural, real, and positive real numbers, respectively. Throughout the paper, \( L_1 \) denotes \( r \times r \) identity matrix and \( \mathcal{Q} \) is a convex polytope in \( \mathbb{R}^2 \). Moreover, the distribution density function is denoted by \( \phi_i \), where \( \phi_i : \mathcal{Q} \rightarrow \mathbb{R}_+ \) represents the likelihood of an event taking place at any arbitrary point in \( \mathcal{Q} \). The function \( \phi_i \) is assumed to be measurable and absolutely continuous. The Euclidean distance function is denoted by \( \| \cdot \| \), and \( |\mathcal{Q}| \) represents the Lebesgue measure of the convex subset \( \mathcal{Q} \).

2. Underlying optimization problem for team-based partitioning

In this section, the problem of agents deployment is addressed by solving two optimization problems. The first optimization problem is defined and solved to partition the main region into subregions assigned to the teams. This is followed by the second optimization problem that is solved inside each team.

2.1. General problem statement

The locational cost function, which is a measure of sensing performance for \( N \) deployed agents, is defined as

\[
H(\mathcal{P}, \mathcal{D}) = \sum_{i=1}^{N} \int_{D_i} f_i(|q| - p_i) \phi_i(q) dq,
\]

where \( D_i \), \( f_i \), and \( p_i \) are respectively the assigned region, the so-called service function and the position of \( i \)th agent, \( \mathcal{D} = \{ \Omega_1, \ldots, \Omega_N \} \) and \( \mathcal{P} = \{ p_1, \ldots, p_N \} \). The set \( \mathcal{D} \) is a collection of...
$N$ closed subsets $\Omega_i$ with disjoint interiors whose union is $\Omega$. The cost function $\mathcal{H}$ is minimized by finding the optimum locations of the agents and their assigned regions $\Omega_i$ whose union is $\Omega$. Obviously, a more effective allocation of agents implies a lower value of sensing cost $\mathcal{H}$.

**Assumption 1.** Throughout this paper, initial locations of the agents are assumed to be distinct, i.e., $p_i \neq p_j$ for $i \neq j$.

Considering that the deployed agents have different sensing capabilities, a different service function $f_i$ and importance function $\phi_i$ are considered for different agents. As expected, sensing performances of the agents should decay as they move away from their location, and hence, sensing performance can be evaluated as a function of the distance from the agent’s location. Therefore, the service function of each agent can be considered as

$$f_i(\|q - p_i\|) = \alpha_i \|q - p_i\|^2,$$

where $\alpha_i \in \mathbb{R}_+$ is a given parameter (referred to as service factor) for heterogeneous agents, $i = 1, \ldots, N$, and $q \in \Omega$. Service factors can be defined with respect to different parameters of interest, such as the energy consumption for performing a task or the travel time. Also, a service factor is assigned to each team corresponding to the capabilities of its member agents relative to other teams.

In this paper, a team-based partitioning is proposed in order to deploy agents over a known environment by exploitng the heterogeneous nature of the agents. This is addressed in the present work by introducing a team-based partitioning of the agents that divides agents into multiple groups pursuing their assigned task. Depending on the coverage requirements, each agent might team up with others based on its density function or complementing capabilities such as communication, dynamics, or sensing. Then, a service function is defined for each team based on the capabilities of its agents. It is noted the number of teams and the agents within each team are decided before deployment. This can be done based on the coverage requirements. For instance, a larger number of agents may be required in certain parts of the region to meet resolution and accuracy requirements of the sensing and coverage tasks. To this end, a team that is expected to cover the assigned area would consist of more agents. Although the structure of teams does not change over time, the agents can be deployed in different teams based on their capabilities. For example, the agents with shorter communication range can be deployed together and cover a smaller region while the agents with longer communication range can cover larger areas. Finally, the control law is applied based on a single integrator dynamics imposed on each agent.

To start with, we define $n$ teams where $t^{th}$ team consists of $n_t$ agents positioned at $p_{t1}, p_{t2}, \ldots, p_{tm}$. Density functions, service factors and assigned region of $m^{th}$ agent in $t^{th}$ team are respectively denoted by $\psi_{tm}$, $\alpha_{tm}$ and $Q_{tm}$.

### 2.2. Two-step optimization problem

The deployment task in the presented team-based framework can be addressed by solving a two-level optimization problem. According to the definition of the teams, the sensing performance measure can be calculated using the following two-level sensing performance measures

$$\mathcal{G}(L, \Omega) = \sum_{t=1}^{n} \int_{Q_t} \alpha_t \|q - l_t\|^2 \psi_t(q) dq,$$

$$\mathcal{G}_t(\mathcal{P}_t, \mathcal{P}_t) = \sum_{m=1}^{n_t} \int_{Q_{tm}} \alpha_{tm} \|q - p_{tm}\|^2 \psi_{tm}(q) dq.$$  \hspace{1cm} (3)

where $l_t$, $\psi_{tm}$ and $\alpha_t$ respectively denote the nucleus, density function, and service factor of $t^{th}$ team as a representative of its team members, and $L = \{l_1, \ldots, l_n\}$, $Q = \{Q_1, \ldots, Q_n\}$, $\mathcal{P}_t = \{p_{t1}, \ldots, p_{tm}\}$ and $\mathcal{P}_t = \{Q_t, \ldots, Q_{tm}\}$. It is assumed that $\psi_{t1} = \psi_{t2} = \cdots = \psi_{tm} = \psi_t$. In this work, we solve the two-level optimization problem of minimizing sensing performance measures (2) and (3) with the objective of developing two appropriate algorithms to collectively deploy heterogeneous agents. The first step is to solve the team-level optimization problem with the cost function (2) to guarantee a locally optimum configuration and partitioning of the convex polytope $\Omega$, while assigning the nuclei of teams as a representative of the locations of team members. The given region is divided into subregions and assigned to teams with respect to their density functions $\psi_t$ and service factors $\alpha_t$, for $t = 1, \ldots, n$. The sensing function in each team represents the collective capabilities of the agents belonging to that team. The present work introduces a new control coverage scheme that can allow deploying teams of heterogeneous agents in order to handle the coverage task in an environment that consists of multiple important regions. Each region is assigned to a team of agents by taking into account their dynamics and sensing capabilities. To this aim, the density function associated with each team may differ from the neighboring teams to reflect the difference in their associated regions of interest. Each agent might team up with others based on its density function, associated dynamics or sensing characteristics. To this purpose, we assume that the agents in the same team have the same density function $\psi_t$. Therefore, the proposed coverage strategy would make it possible to improve reliability, accuracy, and flexibility of the deployment algorithm by taking into account the differences in the embedded sensors and dynamics of agents. In the proposed formulation, the importance functions associated with the neighboring agents may differ from each other. The solution to the problem of minimizing the sensing cost (2) gives a team-level partitioning, where members of the $t^{th}$ team are in charge of covering subregion $Q_t$. Once regions are assigned to teams, solving the second optimization problem with the cost function (3) divides the region assigned to each team among agent members, such that polygons $Q_{t1}, \ldots, Q_{tm}$ are pairwise disjoint interiors whose union covers $Q_t$. It is noted that the two optimization problems are solved sequentially: the team-level optimal solutions are implemented first and then the agents-level optimal solution is applied.

### 2.3. Power diagram and MW-Voronoi diagram

Since regions assigned to teams are divided among their members, allocating a set of convex regions to a team is preferred over non-convex regions. Our previous team-based coverage control algorithms use the standard Voronoi partitioning (Abbasi et al., 2016, 2017b); however, weighted Voronoi partitions are employed in this work to take into account and capitalize on different capabilities of heterogeneous agents in assigning regions to agents.

A Power diagram of $\Omega$ is obtained by assigning an individual weight $\omega_t \in \mathbb{R}_+$ to $t^{th}$ team, for $t = 1, \ldots, n$. Let us define $\mathcal{W} = (\omega_1, \ldots, \omega_n)$ and the Power diagram $\mathcal{V}(\mathcal{T}) = \{V_1, \ldots, V_n\}$ of $\Omega$ with respect to the set of nuclei and weights of teams, $\mathcal{T} = \{(l_1, \omega_1), \ldots, (l_n, \omega_n)\}$, as

$$V_t = \{q \in \Omega | \|q - l_t\|^2 - \omega_t \leq \|q - l_i\|^2 - \omega_i\},$$

where $s = 1, \ldots, n$ and $s \neq t$. Based on the definition (4), the Power cell $V_t$ is a convex region (Okabe et al., 1992). Therefore, the Power diagram $\mathcal{V}(\mathcal{T})$ is a convex partition of $\Omega$. The main advantage of using the Power diagram compared to the Voronoi diagram is that a weight can be assigned to each team representing the capabilities of its members. Subsequently, a team that
includes members with higher capabilities can be assigned to cover a more important region. Additionally, the Power diagram \( V(T) \) has a significant advantage compared to MW-Voronoi in that \( V(T) \) is a convex set.

The obtained Power cells assigned to teams are then divided into a set of convex polytopes to deploy their associated agents. Since each team contains possibly a heterogeneous group of agents, a set of weights \( \mathcal{W}_t = \{\omega_{t1}, \ldots, \omega_{tm}\} \) is given to its member agents to represent their different capabilities. The MW-Voronoi diagram \( V_t(T_t) = \{V_1, V_2, \ldots, V_m\} \) generated with respect to \( T_t = [(p_{t1}, \omega_{t1}), \ldots, (p_{tm}, \omega_{tm})] \) is defined as

\[
V_{tm} = \{q \in V_t | \frac{||q - p_{tm}||}{\omega_{tm}} \leq \frac{||q - p_{tn}||}{\omega_{tn}} \},
\]

where \( s = 1, \ldots, m, s \neq m, m \in \{1, \ldots, n_t\} \) and \( \omega_{tm} \) denotes the weight of \( m \)th agent in \( t \)th team. Because of the properties of the MW-Voronoi diagram, unlike the Power diagram, there would not be any empty cell (Sharifi et al., 2015). If the Power diagram was used to partition the subregion assigned to the team among its agents, an agent may fall outside its own assigned subregion. However, due to the definition of the MW-Voronoi, it is guaranteed that each agent falls inside its assigned subregion. The following remark plays a key role in the development of the main results of our team-based partitioning scheme.

**Remark 1.** It is shown in Sharifi et al. (2015) (Remark 3) that among different partitioning schemes, the center MW-Voronoi configuration is optimal in the sense of minimizing the cost function given in (2). However, when the weights are not constant or even have dynamics, the conventional MW-Voronoi partitioning is no longer optimal. In this case, the problem of deploying heterogeneous agents becomes complicated; however, it can be simplified significantly under an approximation described below. Since the MW-Voronoi partitioning results in non-convex subregions, the MW-Voronoi diagram is approximated by the Power diagram in the team-level partitioning. After dividing the main region among the teams, the region associated with each team is partitioned among members by minimizing the cost function (3). Therefore, an MW-Voronoi configuration is used for the area associated with each team, because the center MW-Voronoi configuration is optimal in the sense of minimizing the defined cost function in (3).

According to Remark 1 and by taking advantage of different weighted Voronoi partitioning methods, a Power diagram and an MW-Voronoi diagram are considered as partitioning schemes for team-level and agent-level partitioning, respectively. Therefore, two-level sensing performances (2) and (3) are defined as follows:

\[
G(T, V) = \sum_{t=1}^{n} \int_{V_t} \alpha_t ||q - l_t||^2 \psi_t(q) dq.
\]

(6)

\[
G(T_t, V_t) = \sum_{m=1}^{n_t} \int_{V_{tm}} \alpha_{tm} ||q - p_{tm}||^2 \psi_{tm}(q) dq.
\]

(7)

The cost function in (6) allows a convex partitioning of the main region into subregions assigned to the teams. Such regions contain only one team because of the service factor, the density function and the weight associated with the team. This is followed by another optimization problem that is solved inside each team. The solution to the second optimization problem (i.e., minimizing (7)) results in deploying the agents in an optimal way inside teams, in which each agent covers the nearest points inside the obtained Power cell, in the sense of the MW-Voronoi definition.

**Remark 2.** Initial polygonal regions assigned to teams are assumed to be nonempty. Also, it is assumed that the initial positions of the agents in each team are inside their assigned polygonal region; these regions are not shared with agents from other teams. These assumptions along with the adaptation law (14) shown later guarantee to assign nonempty cells to the teams. In fact, the adaptation law (14) introduces an inverse relationship between the polar moment of inertia and the weight assigned to each team. For instance, the weight assigned to a team increases when its assigned cell becomes larger according to the definition of the Power cell in (4). Consequently, the proposed adaptive algorithm guarantees assigning nonempty Power cells to teams.

### 3. Development of a team-based weighted partitioning

In this section, a spatially distributed partitioning method is first developed to assign convex subregions of the main area to the teams. Then, the subregion assigned to each team is partitioned among its members using two different methods.

**3.1. Team-level partitioning**

Heterogeneity of agents is the main motivation for deploying agents using the proposed team-based framework. As an extension of definitions in Kantaros et al. (2015), Pierson and Schwager (2016), Pierson et al. (2015) and Pimenta, Kumar, Mesquita, and Pereira (2008), a sensing function is defined for each team to reflect the total energy of its members as

\[
\zeta_t(q, l_t, T_t) = -\sum_{s \neq t} ||q - l_s||^2 - T_t.
\]

(8)

where \( T_t \) is the given sensing performance for \( t \)th team. We call \( T_t \) the performance factor of \( t \)th team that can quantify the capabilities of its agents such as energy constraints on mobility and communication. A team including members equipped with highly accurate sensors is expected to move slower because of dynamic capabilities or accurate measuring. Therefore, it is expected to have a higher performance, but also requires a higher energy consumption to serve any point in the field. The following assumption is important to show a relationship between the performance and the service factor.

**Assumption 2.** The coefficient of the service factor for each team is linearly related to the performance of the team, i.e., \( \alpha_t = \alpha T_t \) where \( \alpha \in \mathbb{R}_{+} \).

**Definition 1.** The difference between the performance factor and the weight of \( t \)th team is called the error factor of the team defined as \( e_t = T_t - \alpha \).

We denote the boundary of the Power cell \( V_t \) by \( \partial V_t \) and the set of teams that share boundaries with \( t \)th team by \( \mathcal{N}_t \). An edge that is shared with neighboring teams \( t \) and \( s \) is shown by \( \partial V_{ts} \). Also, \( N_{ts} \) is the normal vector for the edge of the Power cell \( V_t \), which is shared with another Power cell \( V_s \). Regions assigned to teams are affected by the rate of change of the shared boundary with respect to their nucleus. This is analytically shown in the proof of the next lemma.

**Lemma 3.1.** If \( s \)th team is a neighbor of \( t \)th team, then for any \( q \in \partial V_{ts} \)

\[ \frac{\partial (\partial V_{ts})}{\partial l_t} N_{ts} = \theta_{ts}(l_t, \omega_t, l_s, \omega_s), \]

where

\[
\theta_{ts}(l_t, \omega_t, l_s, \omega_s) = \frac{N_{ts} N_{ts}^T - l_s l_s^T}{||l_t - l_s||^2} \left( l_t + l_s - \frac{q - \omega_t}{\omega_t} \right).
\]
The cost function
\[
\left(\omega_\alpha - \omega_\alpha \|l_h - l_i\| \right) + \frac{1}{2} N_{i,s} + \left(\omega_\alpha - \omega_\alpha \right) \left( l_2 - 2N_{i,s}N_{i,T}^T \right)N_{i,s}
\]

Proof. First, we note that the line to which the points on the team boundaries belong can be described by
\[
\left(N_{i,s}\right)^T \left( q - \frac{l_i + l_h}{2} + \left(\omega_\alpha - \omega_\alpha \right) \left(l_2 - 2N_{i,s}V_{i,s}\right)^T \right) = 0, \quad q \in \partial V_{i,s}.
\]

The normal vector \(N_{i,s}\) associated with \(\partial V_{i,s}\) is obtained by
\[
N_{i,s} = \frac{\left(l_i - l_h\right)}{\left\|l_i - l_h\right\|}.
\]

The partial derivatives of the above two equations with respect to \(l_i\) are respectively calculated as follows
\[
\frac{\partial N_{i,s}}{\partial l_i} \left( q - \frac{l_i + l_h}{2} + \left(\omega_\alpha - \omega_\alpha \right) \left(l_2 - 2N_{i,s}V_{i,s}\right)^T \right) + \left(\frac{\partial \partial V_{i,s}}{\partial l_i} - \frac{1}{2} \left(\omega_\alpha - \omega_\alpha \right) \left(l_2 - 2N_{i,s}V_{i,s}\right)^T \right)N_{i,s} = 0, \quad \left(9\right)
\]
\[
\frac{\partial N_{i,s}}{\partial l_i} = \frac{N_{i,s}N_{i,T}^T \left(l_i - l_h\right)}{\left\|l_i - l_h\right\|}. \quad \left(10\right)
\]

The proof of lemma is completed by substituting \(\left(10\right)\) into \(\left(9\right)\) along with some straightforward algebraic computations.

**Definition 2.** The mass, centroid and polar moment of inertia of the Power cell with respect to the density function of the team are respectively defined as \(M_{V_i} = \int_{V_i} \rho(q) dq\), \(C_{V_i} = \frac{1}{M_{V_i}} \int_{V_i} q \rho(q) dq\) and \(J_{V_i} = \frac{1}{M_{V_i}} \int_{V_i} ||q - l_i||^2 \rho(q) dq\).

**Lemma 3.2.** The cost function \(\left(6\right)\) is continuously differentiable with respect to the nucleus of \(i^{th}\) team and its derivative is
\[
\frac{\partial g(T, \mathcal{V})}{\partial l_i} = 2 \alpha_i M_{V_i} \left(l_i - C_{V_i} - \gamma_i\right),
\]
for \(i \in \left\{1, \ldots, n\right\}\), where
\[
\gamma_i = \frac{1}{\alpha_i M_{V_i}} \sum_{s \in \mathcal{N}_{i,s}} \int_{\partial V_{i,s}} \left( ||q - l_i||^2 \left(\alpha_i \rho(q) - \alpha_i \rho(q)\right) + \left(\omega_\alpha - \omega_\alpha\right) \left(\alpha_i \rho(q) - \alpha_i \rho(q)\right) \right) \partial V_{i,s} dq.
\]

Proof. The derivative of the sensing cost \(g(T, \mathcal{V})\) with respect to the nucleus of \(i^{th}\) team is obtained as
\[
\frac{\partial g(T, \mathcal{V})}{\partial l_i} = \sum_{s = 1}^{n} \int_{\partial V_{i,s}} \alpha_i ||q - l_i||^2 \rho(q) dq + \int_{V_i} \alpha_i ||q - l_i||^2 \rho(q) dq + \sum_{s = 1}^{n} \int_{\partial V_{i,s}} \alpha_i ||q - l_i||^2 \rho(q) \frac{\partial \partial V_{i,s}}{\partial l_i} N_{i,s} dq,
\]
where \(N_{i,s}\) is the normal vector associated with \(\partial V_{i,s}\). According to **Definition 2**, \(\partial g/\partial l_i\) can be rewritten as
\[
\frac{\partial g(T, \mathcal{V})}{\partial l_i} = -2 M_{V_i} \alpha_i \left(C_{V_i} - l_i\right) + \sum_{s = 1}^{n} \int_{\partial V_{i,s}} \alpha_i ||q - l_i||^2 \rho(q) \frac{\partial \partial V_{i,s}}{\partial l_i} N_{i,s} dq. \quad \left(12\right)
\]

As inferred from the definition of the Power diagram, boundaries of the Power cell \(V_i\), that are in the neighborhood of \(i^{th}\) team are dependent on \(l_i\). In other words, the teams whose Power cells do not share any edges with the Power cell associated with \(l_i\), are independent of \(l_i\). This implies that \(\partial \partial V_{i,s}/\partial l_i = 0\) for \(s \notin \mathcal{N}_{i,s}\). Therefore, the last term in \(\left(12\right)\) can be rewritten as
\[
\sum_{s = 1}^{n} \alpha_i \int_{\partial V_{i,s}} ||q - l_i||^2 \rho(q) \frac{\partial \partial V_{i,s}}{\partial l_i} N_{i,s} dq = \sum_{s = 1}^{n} \alpha_i \int_{\partial V_{i,s}} ||q - l_i||^2 \rho(q) \frac{\partial \partial V_{i,s}}{\partial l_i} N_{i,s} dq + \sum_{s = 1}^{n} \alpha_i \int_{\partial V_{i,s}} ||q - l_i||^2 \rho(q) \frac{\partial \partial V_{i,s}}{\partial l_i} N_{i,s} dq + \sum_{s = 1}^{n} \alpha_i \int_{\partial V_{i,s}} ||q - l_i||^2 \rho(q) \frac{\partial \partial V_{i,s}}{\partial l_i} N_{i,s} dq.
\]

The integral on each boundary shared with neighboring teams is the same for teams on both sides except that the normal vectors have opposite sign, i.e., \(N_{i,s} = -N_{i,s}\). In addition, \(\partial \partial V_{i,s}/\partial l_i = \partial \partial V_{j,s}/\partial l_i\), and it can be concluded that
\[
\sum_{s = 1}^{n} \alpha_i \int_{\partial V_{i,s}} ||q - l_i||^2 \rho(q) \frac{\partial \partial V_{i,s}}{\partial l_i} N_{i,s} dq = \sum_{s = 1}^{n} \alpha_i \int_{\partial V_{i,s}} ||q - l_i||^2 \rho(q) \frac{\partial \partial V_{i,s}}{\partial l_i} N_{i,s} dq + \sum_{s = 1}^{n} \alpha_i \int_{\partial V_{i,s}} ||q - l_i||^2 \rho(q) \frac{\partial \partial V_{i,s}}{\partial l_i} N_{i,s} dq.
\]

From the definition of the Power cell in \(\left(4\right)\), we have \(\|q - l_i\|^2 - \omega_\alpha = \|q - l_i\|^2 - \omega_\alpha\) for \(q \in \partial V_{i,s}\). Hence, it is concluded that
\[
\sum_{s = 1}^{n} \alpha_i \int_{\partial V_{i,s}} ||q - l_i||^2 \rho(q) \frac{\partial \partial V_{i,s}}{\partial l_i} N_{i,s} dq = \sum_{s = 1}^{n} \int_{\partial V_{i,s}} \left( \alpha_i ||q - l_i||^2 (\alpha_i \rho(q) - \alpha_i \rho(q)) + (\omega_\alpha - \omega_\alpha) \alpha_i \rho(q) \right) \frac{\partial \partial V_{i,s}}{\partial l_i} N_{i,s} dq. \quad \left(13\right)
\]

Invoking **Lemma 3.1**, the proof is completed by substituting \(\left(13\right)\) into \(\left(12\right)\) and using \(\left(11\right)\).

Next, we propose the following adaptation law to adjust the weights of teams in order to take into account the variations in the corresponding sensing function
\[
\dot{\omega}_i = \frac{k_o}{j_i} \sum_{s \in \mathcal{N}_{i,s}} \int_{\partial V_{i,s}} \left( \xi_i (q, l_i, T_i) - \xi_i (q, l_i, T_i) \right) dq.
\]

where \(k_o \in \mathbb{R}_+\) determines the speed of convergence.

**Lemma 3.3.** The adaptation law to design the team weights is calculated based on the difference between the error factor of a team and its neighboring teams as
\[
\dot{\omega}_i = -\frac{k_o}{j_i} \sum_{s \in \mathcal{N}_{i,s}} ||\partial V_{i,s}|| \left( e_i - e_t \right),
\]

where \(||\partial V_{i,s}||\) is the length of \(\partial V_{i,s}\).

Proof. Assume that \(s^{th}\) team is a neighbor of \(i^{th}\) team. Since \(\|q - l_i\|^2 - \omega_\alpha = \|q - l_s\|^2 - \omega_\alpha\) along the boundary \(\partial V_{i,s}\), the difference between the sensing functions of teams can be simplified as
\[
\xi_i (q, l_i, T_i) - \xi_i (q, l_s, T_s) = (\|q - l_i\|^2 - T_i) - (\|q - l_s\|^2 - T_s) = (T_i - T_s - (\|q - l_i\|^2 - \|q - l_s\|^2)
\]
\[(T_i - T_j) - (\omega_k - \omega_l) = e_i - e_j.\]

Substituting the obtained equality in the equation law (14) results in
\[\dot{\omega}_i = \frac{k_{\omega}}{f_i} \sum_{s \in \mathcal{N}_i} \int_{v_{i,s}} (e_i - e_s) q_{ds} = \frac{-k_{\omega}}{f_i} \sum_{s \in \mathcal{N}_i} |\partial V_{i,s}|(e_s - e_i),\]
and this completes the proof of lemma.

**Lemma 3.4.** The weights \(\omega_i\) obtained by applying the adaptation law (14) are bounded and
\[\lim_{\tau \to \infty} e_i(\tau) - e_j(\tau) = 0, \quad \forall i, j \in \{1, \ldots, n\}.

**Remark 3.** Since the error factors of the teams \(e_i\) converge to the same value according to Lemma 3.4, there is a positive constant \(c\) so that weights \(\omega_i\) converge to \(T_i - c\) for any \(i \in \{1, \ldots, n\}\).

We note that the locations of agent members in each team are represented by the nucleus of that team. The following dynamics is enforced on the nucleus
\[l_i = -k_i(l_i - C_{V_i} - \gamma_i), \quad (16)\]
where \(k_i\) is a positive gain.

**Theorem 3.5.** The overall teams asymptotically converge to a local minimum by applying the controller in (14) and (16). That is, for \(i \in \{1, \ldots, n\}\),
\[\lim_{\tau \to \infty} l_i(\tau) = C_{V_i} + \gamma_i.

**Proof.** Consider a Lyapunov function candidate as \(V = \varphi(T, V)\). The Lyapunov function is obviously lower bounded by zero because of the cost function (6). Employing Assumption 2 and Remark 3, it is concluded that
\[\frac{\partial \varphi(T, V)}{\partial \omega_i} = \frac{\partial}{\partial \omega_i} \sum_{k=1}^{n} \int_{v_{i,k}} \alpha(\omega_k + c) |q - l_i|^2 \varphi_k(q) dq = -\alpha \int_{v_i} |q - l_i|^2 \varphi(q) dq = -\alpha f_{V_i}.

The time derivative of \(V\) is
\[\frac{dV}{dt} = \sum_{t=1}^{n} \left( \frac{\partial l_i}{\partial \varphi} \frac{\partial \varphi(T, V)}{\partial l_i} + \frac{\partial L_i}{\partial \varphi} \frac{\partial \varphi(T, V)}{\partial L_i} \right).
\]
Substituting (16) and (15) into the time derivative of \(V\) yields
\[\dot{V} = \sum_{t=1}^{n} (\alpha M_{V_i}(l_i - C_{V_i} - \gamma_i)^T - k_i(l_i - C_{V_i} - \gamma_i)) - \sum_{t=1}^{n} |\partial V_{i,s}|(e_s - e_i) = -\sum_{t=1}^{n} \alpha M_{V_i}k_i l_i - C_{V_i} - \gamma_i|^2.
\]
Since \(M_{V_i}, k_i \in \mathbb{R}_{+}\), \(\dot{V}\) is negative semidefinite. As shown in Schwager, Rus, and Slotine (2009), \(V\) is uniformly bounded which results in the uniform continuity of \(V\). Next, due to the boundedness of \(V\) and the continuity of \(V(\tau)\), it is proven by Barbalat’s lemma (Khalil, 2002) that
\[\lim_{\tau \to \infty} V(\tau) = 0 \Rightarrow \lim_{\tau \to \infty} |T_i M_{V_i} k_i l_i - C_{V_i} - \gamma_i|^2 = 0.

We note that \(t\) is used to refer to the team number, whereas \(r\) denotes the continuous-time index.

**Remark 4.** Since teams are composed of heterogeneous agents, the center of mass of region assigned to each team is not optimal in the sense of minimizing the cost function (6). To coordinate teams of heterogeneous agents with different capabilities for serving multiple regions of interest, the control strategy (16) places the nuclei of teams near the center of mass of assigned regions. It is noted that when all teams have agents with the same service factor and density factor, the control strategy (16) coincides with the centroidal configuration. In other words, if \(\alpha_1 = \cdots = \alpha_n\) and \(\varphi_1 = \cdots = \varphi_n\), then \(\gamma_1 = \cdots = \gamma_n = 0\), and consequently \(\lim_{\tau \to \infty} l_i(\tau) = C_{V_i}\) for \(i \in \{1, \ldots, n\}\).

3.2. Agent-level partitioning

The main region was first partitioned among teams based on the capabilities of their agent members. Then, the regions assigned to teams need be divided among their members. Since the area assigned to a team moves based on the underlying dynamics of its nucleus, associated agents also need to adapt and move accordingly. We propose two strategies for the agent-level partitioning task. We restrict our attention to the MW-Voronoi diagrams due to their advantages discussed before. In the first algorithm, weights of agents are assumed to be constant. Therefore, the weights are defined based on the capabilities of the agents in order to achieve the centroidal MW-Voronoi configuration. In the second algorithm, given the fact that in practice agents perform under different situations in the environment, their weights might be dependent on how well they perform the assigned task. Therefore, an adaptive control law is proposed to update weights of agents with respect to their performance.

**Definition 3.** The basic characteristics of the MW-Voronoi partitions including their associated mass, centroid and polar moment of inertia for an MW-Voronoi cell are defined as \(M_{V_{im}} = \int_{V_{imm}} q \varphi_i(q) dq\), \(C_{V_{im}} = \frac{1}{M_{V_{imm}}} \int_{V_{imm}} q \varphi_i(q) dq\), and \(J_{V_{im}} = \int_{V_{imm}} ||q - p_{im}||^2 \varphi_i(q) dq\).

**Case 1** Heterogeneous members with constant weights: Suppose that the service factors for agents are given and constant. The following result is related to the distributed deployment for a network of heterogeneous mobile agents with different service costs (Sharifi et al., 2015).

**Theorem 3.6.** Partitioning the subregion assigned to each team among its members yields a centroidal MW-Voronoi configuration inside the team if \(\omega_{nim} = \frac{1}{\sqrt{\omega_{nim}}}\) and the following single integrator dynamics is enforced on the agents
\[\dot{p_{im}} = -k_m(p_{im} - C_{V_{im}}), \quad (17)\]
where \(k_m \in \mathbb{R}_+\).

**Proof.** Consider a Lyapunov function candidate as \(V = \varphi(T, V) = \sum_{m=1}^{m} \int_{V_{imm}} \alpha_m ||q - p_{im}||^2 \varphi_i(q) dq\), which is lower bounded by zero. Therefore,
\[\dot{V} = \alpha_m \sum_{m=1}^{m} \int_{V_{imm}} (q - p_{im})^T \varphi_i(q) \dot{p_{im}} dq = -\alpha_m \sum_{m=1}^{m} M_{V_{imm}} k_m ||p_{im} - C_{V_{im}}||^2.
\]
which is negative semidefinite. The centroidal MW-Voronoi configuration is then concluded due to Proposition 1 and Remark 3 in Sharifi et al. (2015).

(Case 2) Heterogeneous members operating with different sensing functions: Since teams consist of heterogeneous agents with different capabilities and each agent possibly operates in a different region, agents have therefore different sensing functions. After assigning subregions to teams based on the energy resources of their members in the team-level partitioning, heterogeneous sensing capabilities of agents in each team play a key role in partitioning the team’s assigned subregion among their members. Consequently, the proposed hierarchical team-based framework makes it possible to incorporate heterogeneous energy resources and sensing capabilities of agents in the deployment problem. The sensing function of \( m \)th agent in \( t \)th team is considered as

\[
\zeta_m(q, p_m, T_m) = -\frac{\|q - p_m\|^2}{T_m},
\]

where \( T_m \) is assigned based on the sensing capabilities of the agent. An agent with a better sensing capability offers a higher performance that comes with a higher service cost. The following assumption represents this statement.

**Assumption 3.** The service factor of each agent is linearly related to the performance of the agent, i.e., \( \alpha_{tm} = a_t T_m \), where \( a_t \in \mathbb{R}_+ \), for \( t \in \{ 1, \ldots, n \} \) and \( m \in \{ 1, \ldots, n_t \} \).

**Definition 4.** The ratio of the performance with respect to the weight of each agent is called the error factor of the agent and defined as \( e_{tm} = \frac{\alpha_{tm}}{T_m} \), for \( t \in \{ 1, \ldots, n \} \) and \( m \in \{ 1, \ldots, n_t \} \).

We denote by \( N_{tm} \) the set of agents belonging to the \( t \)th team that share boundaries with \( m \)th agent. An edge that is shared with neighboring agent \( m \) and agent \( s \) in \( t \)th team is shown by \( \partial V_{tm,s} \). Also, \( N_{tm,s} \) denotes the normal vector associated with the edge \( \partial V_{tm,s} \).

Next, we propose following adaptation law to adjust the weights of agents in order to take into account variations in the sensing functions

\[
\dot{\omega}_{tm} = \frac{k_{im}}{T_{im}} \sum_{t \in N_{tm,s}} \int_{\partial V_{tm,s}} \frac{\omega_{tm}(q, p_m, T_m)}{\|q - p_m\|^2} \zeta_m(q, p_m, T_m) dq - \zeta_m(q, p_m, T_m),
\]

where \( k_{im} \in \mathbb{R}_+ \). A weight is adjusted by comparing the agent’s sensing performance with those of its neighbors according to the proposed control law. For time-varying environments (Pierson et al., 2015; Pierson, Figueiredo, Pimenta, & Schwager, 2017; Pierson & Schwager, 2016; Zuo, Shi, & Yan, 2019), an adaptive control law is proposed to update the weights of agents and teams with respect to their performance.

**Lemma 3.7.** The difference between the error factor of the agent with those of its neighbors determines the adaptation law as

\[
\dot{\omega}_{tm} = -\frac{k_{im}}{T_{im}} \sum_{t \in N_{tm,s}} |\partial V_{tm,s}| (e_{tm} - e_t).
\]

**Proof.** Suppose that \( s \)th agent is a neighbor of \( m \)th agent in \( r \)th team, i.e., \( ts \in N_{tm} \). Since \( \alpha_{ts} = \alpha_{tm} |q - p_m| = \alpha_{ts} |q - p_s| \) along the boundary \( \partial V_{tm,s} \), the difference between the sensing functions of the neighbor agents can be simplified as

\[
\zeta_m(q, p_m, T_m) - \zeta_s(q, p_s, T_s) = \frac{\|q - p_m\|^2}{T_s} - \frac{\|q - p_m\|^2}{T_m}.
\]

Substituting the above into the adaptation law (19) results in

\[
\dot{\omega}_{tm} = \frac{k_{im}}{T_{im}} \sum_{t \in N_{tm,s}} \int_{\partial V_{tm,s}} (e_{tm} - e_t) dq = -\frac{k_{im}}{T_{im}} \sum_{t \in N_{tm,s}} |\partial V_{tm,s}| (e_{tm} - e_t).
\]

and this completes the proof.

**Lemma 3.8.** The weights \( \omega_{tm} \) are bounded by applying the adaptation law in (20) and

\[
\lim_{t \to \infty} e_{tm}(\tau) - e_t(\tau) = 0,
\]

for \( t \in \{ 1, \ldots, n \} \) and \( m \in \{ 1, \ldots, n_t \} \).

**Proof.** The adaptation law (20) can be rewritten in terms of \( e_{tm} \) as

\[
\dot{e}_{tm} = -\frac{k_{im}}{T_{im}} \sum_{t \in N_{tm,s}} |\partial V_{tm,s}| (e_{tm} - e_t).
\]

The obtained controller can be described in a vector form as

\[
\dot{E}_t = -M_t L_t E_t,
\]

where

\[
E_t = \begin{bmatrix}
e_{t1} \\
\vdots \\
e_{tn}
\end{bmatrix},
\]

\[
M_t = \begin{bmatrix}
k_{im} & 0 & 0 \\
T_{im} & 0 & 0 \\
0 & 0 & 0
\end{bmatrix},
\]

\[
L_t = \begin{cases}
-|\partial V_{tm,s}|, & \text{if } s \neq m, ts \in N_{tm} \\
0, & \text{otherwise}
\end{cases}
\]

Matrix \( L_t \) above acts like the Laplacian matrix of the weighted graph. Matrix \( M_t L_t \) is a positive semi-definite matrix, since the Laplacian matrix is positive semi-definite and \( M_t \) is positive definite. Therefore, based on the properties of a stable linear filter (Bullo et al., 2009), it is concluded that \( \lim_{t \to \infty} \dot{E}_t(\tau) = 0 \), and hence \( L_t E_t = 0 \). Since \( L_t \) is the Laplacian matrix, it is concluded that \( e_{tm} = e_t = 1/c_t \) where \( c_t \) is a constant positive according to Pierson and Schwager (2016). Consequently, \( T_{im} = e_t \alpha_{tm} \), for \( t \in \{ 1, \ldots, n \} \). Finally, next theorem provides proof of convergence when the adaptation law (19) is implemented.

**Theorem 3.9.** Suppose that Assumptions 1 and 3 are satisfied. Agents in the \( t \)th team then converge to a local minimum by applying the control laws (17) and (19). That is

\[
\lim_{t \to \infty} p_{tm}(\tau) = C_{tm},
\]

for \( t \in \{ 1, \ldots, n \} \) and \( m \in \{ 1, \ldots, n_t \} \).

**Proof.** The Lyapunov function is defined by considering Assumption 3 as

\[
V = G_t(T_t, \dot{V}_t) = a_t \sum_{m=1}^{n_t} \int_{V_m} T_m \|q - p_m\|^2 \phi_t(q) dq.
\]
The above Lyapunov function candidate is obviously lower bounded by zero. Due to the implication of Lemma 3.8, i.e., \( T_m = c_t \omega_m \), the Lyapunov function candidate can be rewritten as

\[
V = a_t c_t \sum_{m=1}^{n_t} \int_{V_m} \omega_m \| q - p_m \| \psi(q) dq.
\]

The time derivative of \( V \) is obtained as

\[
\dot{V} = a_t c_t \sum_{m=1}^{n_t} \int_{V_m} \omega_m (q - p_m)^\top \psi(q) \partial V_m dq = a_t c_t \sum_{m=1}^{n_t} \int_{V_m} \| q - p_m \|^2 \psi(q) \omega_m dq.
\]

Substituting (16) and (14) into the above time derivative of \( V \) yields

\[
\dot{V} = a_t c_t \sum_{m=1}^{n_t} \omega_m (M_{V_m}(p_m - C_{V_m}))^\top \\
\left(-k_m(p_m - C_{V_m}) - a_t c_t \sum_{m=1}^{n_t} k_m \sum_{m=1}^{n_t} |\partial V_m, e_m| (e_m - e_m)\right) \\
= - \sum_{m=1}^{n_t} a_t c_t M_{V_m} k_m \| (p_m - C_{V_m}) \|^2.
\]

Since \( a_t c_t, M_{V_m}, k_m \in \mathbb{R}_+ \), \( \dot{V} \) is negative semi-definite. The rest of the proof follows a similar argument as in the proof of Theorem 3.5.

It is noted that the agents communication is done through the implementation of a greedy algorithm as proposed in Abbasi et al. (2016).

4. Simulation results and discussion

Performance of the proposed coverage control algorithms of this paper is validated via numerical examples that represent different scenarios. To this end, five teams of agents are deployed in a region where each team has its own assigned region of interest, which represents heterogeneity in the assigned coverage tasks.

Simulation study I

In the first example, the teams are assumed to have different number of agents as shown in Fig. 1. An example of this scenario is when it is required to have control over the number of agents deployed in different parts of the region – one immediate application is where the agents have different capabilities, and each of them is required to cover certain parts of a given region. This can be due to heterogeneous sensing capabilities and sensing (coverage) task defined by various importance functions. The corresponding importance function is shown in Fig. 3. The traditional deployment methods in the literature are unable to distinguish between agents based on their potential heterogeneity throughout the deployment. This is addressed by implementing the proposed team-based coverage control method. Fig. 1 shows the agents’ initial and final configurations along with their traversed path for the proposed deployment approach. In this example, it is assumed that the teams and agents are deployed using a set of weights as an indication of heterogeneity in their sensing capabilities. The agent weights are set to be constant and known a priori throughout the deployment (as described in case 1 of the previous section) while the team weights are adaptively adjusted. The weights corresponding to the agents that belong to each team are assumed to be [0.43, 0.41, 0.36, 0.33], [0.42, 0.4, 0.35, 0.34, 0.3], [0.41, 0.37, 0.36, 0.32, 0.31], [0.28, 0.24, 0.17, 0.15] and [0.5, 0.48, 0.44]. The differences in the performance of the agents within each team would remain unchanged as agents spread over the region to perform the underlying coverage task. The initial weights associated with the teams shown by red, yellow, green, blue and purple are considered to be all 1, while those weights after convergence are respectively 1.5, 1.2, 1.2, 0.1, 0.2 using the adaptive control law (14).

Simulation study II

In the second example, an adaptive Power diagram-based coverage approach is implemented as proposed in the previous sections. The presented algorithm is a two-level partitioning method where agents are deployed as the combination of heterogeneous teams and agents. Two different types of coverage algorithms are implemented, where the team-level coverage is carried out via Power diagrams while a multiplicative (MW-Voronoi) partitioning is used to address heterogeneity in the agents level. Power diagrams that obtain regions assigned to each team are representative of differences in the collective capabilities of teams of agents and provide convex polytopes for the agent-level partitioning. Then, the differences in dynamics and sensing capabilities of agents compared to other agents in the same team are taken into account via the MW-Voronoi partitioning. The associated importance function is shown in Fig. 4. In this example, it is assumed that weights of teams and also agents belonging to each team are changing to finally converge to their representative performance factor decided based on the sensing capability. In other words, in contrast to the previous example, we relax the assumption that the agents performance factor is
known *a priori*. Instead, two adaptive laws are implemented on the agent- and team-level partitioning to ensure that the weights would converge to their representative performance factor. This can be interpreted as an online performance evaluation approach. The dynamics on weights are governed by the proposed adaptive team-based weighted control law in (14). In this example, initial weights for five teams are assumed to be $[\omega_1 \ \omega_2 \ \omega_3 \ \omega_4 \ \omega_5] = [1 \ 1 \ 1 \ 1 \ 1]$. The associated sensing performance of the teams are assigned as $[T_1 \ T_2 \ T_3 \ T_4 \ T_5] = [1.5 \ 1.4 \ 1.2 \ 1 \ 1.1]$. Teams with higher weights are expected to take a larger area compared to their neighbors. The area that each team covers evolves according to the dynamics imposed on its weight to ensure that the difference between weights (that represents the relative performance) finally converges to the difference in their performance values. The area associated with each team is shown with a different color in Fig. 2, where red, yellow, green, blue and purple represent first to fifth teams, respectively. It is noted that the weights converge first because without weights converging to their final values, the region assigned to each team and each agent would change, which causes the nucleus and centroid to vary. This leads to the agents moving towards their new centroid which means that the agents cannot converge before convergence of the assigned weights. The final converged values of the weights associated with teams shown by red, yellow, green, blue and purple are $[1.26 \ 1.16 \ 0.96 \ 0.76 \ 0.86]$, respectively. Fig. 5 shows the convergence of the weights associated with the teams.

In addition to the team performance that is taken into account, we utilize an adaptive multiplicative Voronoi-based partitioning for agents belonging to each team. As shown in Fig. 2, the agents take different shares from the regions assigned to their team as an indication of heterogeneity in their relative performance. The heterogeneity in agents performance can be found in the relative difference in their sensing performance. For example, in case of visual sensing capabilities, different agents may provide different sensing qualities due to the camera defects or a condition caused by their environment like dust on their lenses. In the present work, the proposed adaptive framework ensures that multiplicative weights for the agents inside each team can also
This paper presents a team-based coverage control method for deploying a group of heterogeneous agents to cover multiple regions of interest. The number of teams and their members are determined based on the number of regions of interest and their associated degree of importance. The main region is first partitioned into multiple subregions among teams with respect to the regions of interest. Considering the fact that teams can include different agents with possibly complementary capabilities,
the Power diagram is used for team-level partitioning aiming at assigning larger regions to teams that include agents with higher capabilities. MW-Voronoi diagrams are then used for agent-level partitioning in order to not only guarantee collision avoidance, but also to assign subregions to agents based on their capabilities. Weights associated with both teams and agents are determined according to adaptation laws that are obtained using associated sensing functions. The proposed team-based weighted partitioning methods of this paper provide a practical tool for deploying (even a large group of) agents around multiple regions of interest by taking their heterogeneity into account.

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