Adaptive Sliding Mode Fault Diagnosis for LPV Systems with Uncertain Scheduling Variables

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Abstract— **This paper presents an adaptive sliding mode observer for actuator fault diagnosis of linear parameter-varying (LPV) systems with imperfectly measured scheduling variables due to noisy or faulty measurements. The developed adaptive observer is able to cope with bounded uncertainties and faults without a priori knowledge of their bounds. Moreover, the actuator fault can be estimated from the principle of equivalent control after minimizing the effect of the scheduling variable uncertainties. The performance of the proposed design approach is validated using simulation studies.**

I. Introduction

Fault detection and isolation (FDI) is a critical task aiming at ensuring safety conditions for industrial systems. With accurate FDI algorithms and fault tolerant controllers, the fault negative impact on the system can be alleviated or even eliminated. FDI filters designed based on linear models usually have the desired performance around a specific operating point. However, the system might encounter a performance degradation as it moves away from the operating point, which can lead to false alarms or even missing fault detection. Within the linear parameter-varying (LPV) framework, a class of nonlinear/time-varying systems can be represented in terms of parametrized linear dynamics in which the model coefficients depend on a set of measurable variables called *scheduling variables*. Therefore, FDI filters can be developed for nonlinear/time-varying systems [1], [2] by extending the FDI filter design approaches of linear time-invariant (LTI) systems. The LPV filters are automatically scheduled based on the measured scheduling variables [3], [4] in such a way to guarantee the stability and the required performance over a wide range of operation [5]–[7].

In the literature, several techniques have been developed for fault diagnosis of LPV systems such as the norm-based robust optimization filters [8], [9], geometric approach [10], inversion based methods [11], eigenstructure assignment [12], unknown input observers [13] and sliding mode observers [14]–[16]. However, the scheduling variables are assumed to be exactly known or perfectly measurable. In realistic situations, however, scheduling variables are subject to uncertainties due to several factors such as measurement noises, inaccurate sensors, and faults that may occur in the sensors. Such uncertainties surely affect the state estimation, as well as the FDI results, which may eventually lead to false alarms or missing detections.

A few research works have addressed the observer design problems with uncertain scheduling parameters for LPV systems. In [17], an LPV observer proposed for state estimation, was able to cope with the scheduling variable uncertainty by means of linear matrix inequalities (LMIs). Combined controller-observer designs were presented in [18], [19] for uncertain LPV systems. More recently, in [20] an H_∞ LPV filter has been proposed with additive as well as multiplicative uncertainties in the measurements. For FDI problem, in [21], a robust $\mathcal{H}_\infty$ sliding mode observer was developed for LPV systems subject to uncertainties on scheduling variables. Robust and adaptive fault detection method based on multi-model approach was explored for a class of parameter uncertain LPV systems in [22]. In [23], a robust sliding mode observer was developed for reconstructing faults of uncertain LPV systems with imperfect knowledge of the scheduling variables.

Sliding mode design approach is characterized by a guaranteed convergence and a high robustness against plant-model mismatch, uncertainties, and faulty sensor measurements. Therefore, sliding mode observer is a powerful approach to consider for LPV systems and in this paper, we propose an adaptive sliding mode technique for fault detection and isolation of LPV systems. Comparing to [23] where the observer gain is selected to be higher than the uncertainties and faults upper bounds, the main advantage of this work is that the observer gain is continuously adapted to overcome the system uncertainties and faults without any a priori knowledge about their bounds [24]–[27]. The obtained gain has a sufficient value leading to stability while reducing the chattering phenomenon. Moreover, the FDI procedure can no longer directly estimate the actuator faults as in the case of perfectly known scheduling variables [27] due to the presence of uncertain terms. After analyzing the *equivalent output estimation error injection* concept [28], [29], the fault can be deduced from the equivalent control after minimizing the effect of the uncertainties and faulty measurements on the error of the estimated fault [8]. Therefore, unlike the work in [22], where the fault is detected by a set of filters, the sliding mode observer enables to detect, identify, and reconstruct the real profiles of the faults in addition to its adaptive features.

*This publication was made possible by NPRP grant No. 5-574-2-233 from the Qatar National Research Fund (a member of Qatar Foundation). The statements made herein are solely the responsibility of the authors.

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The paper is organized as follows. LPV system analysis and preliminaries are presented in Section 2. An LPV adaptive sliding mode observer design is then developed for LPV models in Section 3. In Section 4, the proposed observer design method is validated via a simulation example. Finally, concluding remarks are made in Section 5.

Notation: In this paper, $\mathbb{R}^n_+$ represents the set of vectors in $\mathbb{R}^n$ with positive elements, $|.|$ denotes the absolute value and $||.||$ represents the Euclidean norm. Finally, I_p is a $p \times p$ identity matrix.

II. LPV SYSTEM ANALYSIS

Consider an LPV system described by

$$\begin{aligned} \dot{x}(t) &= A(\theta(t))x(t) + B(\theta(t))u(t) + Ew(t) + Hf_a(t), \\ y(t) &= Cx(t), \end{aligned} \tag{1}$$

where the state vector is $x \in \mathbb{R}^n$, the input vector is $u \in \mathbb{R}^m$, and the output vector is $y \in \mathbb{R}^p$. The matrices $A(\theta(t)) \in \mathbb{R}^{n\times n}$ and $B(\theta(t)) \in \mathbb{R}^{n\times m}$ are continuous functions depending on the scheduling variables $\theta(t) \in \mathbb{R}^l$, whereas $C \in \mathbb{R}^{p\times n}$ with $q \leq p < n$, $E \in \mathbb{R}^{n\times r}$ and $H \in \mathbb{R}^{n\times q}$ are assumed to be known constant matrices. The vector $w(t)$ represents all modeling uncertainties and disturbances and the function $f_a(t) \in \mathbb{R}^q$ denotes the actuator fault experienced by the system. In order to simplify the notations, θ is used to denote the time-varying scheduling variables $\theta(t)$ and θ_i is the i^{th} element of θ. It is assumed that the inputs and the outputs of the system are bounded after the occurrence of the actuator fault.

Remark 1: In general LPV systems, the matrix H can also depend on the scheduling variable θ. It is assumed here that in such a case, it can be factorized as $H(\theta) = HF(\theta)$ where $H \in \mathbb{R}^{n\times q}$ is fixed and $F(\theta) \in \mathbb{R}^{q\times q}$ is a parameter-varying invertible matrix. By this change of variable, one will have a constant matrix H and the new fault is considered as $f_b(t) = F(\theta)f_a(t)$. The observer design strategy will initially estimate $f_b(t)$ and then $f_a(t)$ will be obtained as $f_a(t) = F^{-1}(\theta)f_b(t)$.

The variable $\theta(t)$ is defined over a compact set $P_\theta \subset \mathbb{R}^l$ such that $\theta(t) : \mathbb{R} \to \mathbb{P}_\theta$. P_θ considered as a polytope is defined as the convex hull given by M vertices θ_{v_i} where $M = 2^l$, is formed by only considering the bounds of θ_i for $i = 1, \ldots, l$. Since the matrices in (1) depend affinely on θ that can be expressed as a convex combination of M vertices θ_{v_i}, each matrix, denoted as Q, is represented in a polytopic LPV form as

$$Q(\theta) = \sum_{i=1}^{M} \mu_i Q(\theta_{\mathrm{v}_i}), \tag{2}$$

in which $\sum_{i=1}^{M} \mu_i = 1$ with $\mu_i \geq 0$.

Consequently, it is assumed that rank$(C\,H) = q$, and the system (1) can be rewritten as [30]

$$\begin{aligned} \dot{x}_1(t) &= A_{11}(\theta)x_1(t) + A_{12}(\theta)x_2(t) + B_1(\theta)u(t) + E_1w(t), \\ \dot{x}_2(t) &= A_{21}(\theta)x_1(t) + A_{22}(\theta)x_2(t) + B_2(\theta)u(t) + E_2w(t) \\ &\quad + H_2f_a(t), \\ y(t) &= C_2x_2(t), \end{aligned} \tag{3}$$

where $x_1 \in \mathbb{R}^{n-p}$, $x_2 \in \mathbb{R}^p$, $C_2 \in \mathbb{R}^{p\times p}$ is a nonsingular matrix, and $H_2 = \begin{bmatrix} 0_{(p-q)\times q} \\ H_{22} \end{bmatrix}$ with $H_{22} \in \mathbb{R}^{q\times q}$ of full rank.

Moreover, it is assumed that there exists a transformation

$$T = \begin{bmatrix} I_{(n-p)} & L \\ 0 & I_p \end{bmatrix}, \tag{4}$$

where $L \in \mathbb{R}^{(n-p)\times p}$ has the following structure

$$L = \begin{bmatrix} L_1 & 0_{(n-p)\times q} \end{bmatrix}, \tag{5}$$

with $L_1 \in \mathbb{R}^{(n-p)\times(p-q)}$ such that $A_{11}(\theta) + LA_{21}(\theta)$ is stable over all the trajectories of θ. Moreover, L is computed later on in order to ensure the robustness of the fault estimation towards different uncertainties encountered by the system.

The system (3) has the following form in the new coordinate system $z = Tx$:

$$\begin{aligned} \dot{z}_1(t) &= \tilde{A}_{11}(\theta)z_1(t) + \tilde{A}_{12}(\theta)z_2(t) + \tilde{B}_1(\theta)u(t) + \tilde{E}_1w(t), \\ \dot{z}_2(t) &= \tilde{A}_{21}(\theta)z_1(t) + \tilde{A}_{22}(\theta)z_2(t) + \tilde{B}_2(\theta)u(t) + \tilde{E}_2w(t) \\ &\quad + H_2f_a(t), \\ y(t) &= C_2z_2(t), \end{aligned} \tag{6}$$

where

$$\begin{aligned} &\begin{bmatrix} \tilde{A}_{11}(\theta) & \tilde{A}_{12}(\theta) \\ \tilde{A}_{21}(\theta) & \tilde{A}_{22}(\theta) \end{bmatrix} = T \begin{bmatrix} A_{11}(\theta) & A_{12}(\theta) \\ A_{21}(\theta) & A_{22}(\theta) \end{bmatrix} T^{-1}, \\ &\begin{bmatrix} \tilde{B}_1(\theta) \\ \tilde{B}_2(\theta) \end{bmatrix} = T \begin{bmatrix} B_1(\theta) \\ B_2(\theta) \end{bmatrix} \text{ and } \begin{bmatrix} \tilde{E}_1(\theta) \\ \tilde{E}_2(\theta) \end{bmatrix} = T \begin{bmatrix} E_1(\theta) \\ E_2(\theta) \end{bmatrix}. \end{aligned} \tag{7}$$

Since the matrices in (1) depend affinely on θ, the matrices in (6) are also continuous functions that depend affinely on θ.

In this paper, we consider that the scheduling variables θ are not perfectly measured or estimated. The uncertain scheduling vector is denoted by $\hat{\theta}$ and it is assumed that $\theta^e = \hat{\theta} - \theta$ is bounded; $\hat{\theta}$ is thus defined over a compact set $P_{\hat{\theta}} \subset \mathbb{R}^l$ considered as a polytope defined by $M = 2^l$ vertices based on the bounds of $\hat{\theta}$.

After analyzing and reformulating the LPV model as in (6), the design of an LPV adaptive sliding mode observer is addressed in the next section.

III. ADAPTIVE SLIDING MODE OBSERVER FOR LPV SYSTEMS

In this paper, the proposed observer depends on the uncertain scheduling vector $\hat{\theta}$ as follows

$$\begin{aligned}\dot{\hat{z}}_1(t) &= \tilde{A}_{11}(\hat{\theta})\hat{z}_1(t) + \tilde{A}_{12}(\hat{\theta})C_2^{-1}y(t) + \tilde{B}_1(\hat{\theta})u(t),\\ \dot{\hat{z}}_2(t) &= \tilde{A}_{21}(\hat{\theta})\hat{z}_1(t) + \tilde{A}_{22}(\hat{\theta})\hat{z}_2(t) + \tilde{B}_2(\hat{\theta})u(t)\\ &\quad - K\big(\hat{y}(t) - y(t)\big) + C_2^{-1}\nu(t),\\ \hat{y}(t) &= C_2\hat{z}_2(t),\end{aligned} \tag{8}$$

where K is the observer gain to be designed. The p components $\nu_j(t)$ of the observer function $\nu(t)$ are of the form

$$\nu_j(t) = -\lambda_j(t)\ \ \mathrm{sign}\big(\hat{y}_j(t) - y_j(t)\big), \tag{9}$$

where $\hat{y}_j(t)$ and $y_j(t)$ are the j^{th} entry of the vector $\hat{y}(t)$ and the output $y(t)$, respectively. Each gain $\lambda_j(t)$ of the vector $\lambda(t) \in \mathbb{R}^p$ is defined as in [31]

$$\dot{\lambda}_j(t) = \bar{\lambda}_j\ \big|\hat{y}_j(t) - y_j(t)\big|, \tag{10}$$

with $\lambda_j(0) > 0$ and $\bar{\lambda}_j > 0$ for $j = 1, \ldots, p$.

According to (10), the observer gain $\lambda(t)$ will be increasing until the sliding motion starts around $e_y(t) = 0$. The gain will be keeping its value after reaching the sliding surface. Consequently, the induced chattering is reduced because the gain of the discontinuous signal $\nu(t)$ is not overestimated while no *a priori* knowledge of the uncertainty and perturbation bounds is required.

Defining the observation errors as $e_1(t) = \hat{z}_1(t) - z_1(t)$ and $e_y(t) = \hat{y}(t) - y(t)$, the error dynamic with respect to (6) and (8) is then obtained as follows

$$\begin{aligned}\begin{bmatrix}\dot{e}_1(t)\\ \dot{e}_y(t)\end{bmatrix} &= \begin{bmatrix}\tilde{A}_{11}(\hat{\theta}) & 0\\ C_2\tilde{A}_{21}(\hat{\theta}) & C_2\tilde{A}_{22}(\hat{\theta})C_2^{-1} - C_2K\end{bmatrix}\begin{bmatrix}e_1(t)\\ e_y(t)\end{bmatrix}\\ &\quad + \begin{bmatrix}\tilde{A}^e_{11}(\hat{\theta},\theta) & 0\\ C_2\tilde{A}^e_{21}(\hat{\theta},\theta) & C_2\tilde{A}^e_{22}(\hat{\theta},\theta)\end{bmatrix}\begin{bmatrix}z_1(t)\\ z_2(t)\end{bmatrix}\\ &\quad + \begin{bmatrix}\tilde{B}^e_1(\hat{\theta},\theta)\\ C_2\tilde{B}^e_2(\hat{\theta},\theta)\end{bmatrix}u(t) + \begin{bmatrix}\tilde{A}^e_{12}(\hat{\theta},\theta)C_2^{-1}\\ 0\end{bmatrix}y(t)\\ &\quad - \begin{bmatrix}\tilde{E}_1\\ C_2\tilde{E}_2\end{bmatrix}w(t) - \begin{bmatrix}0\\ C_2H_2\end{bmatrix}f_a(t) + \begin{bmatrix}0\\ 1\end{bmatrix}\nu(t),\end{aligned} \tag{11}$$

where the error system state-space matrices of the form $\tilde{Q}^e_{ij}$ are defined as $\tilde{Q}^e_{ij}(\hat{\theta},\theta) = Q_{ij}(\hat{\theta}) - Q_{ij}(\theta)$, and $\tilde{B}^e_i(\hat{\theta},\theta) = \tilde{B}_i(\hat{\theta}) - \tilde{B}_i(\theta)$ for $i, j \in \{1, 2\}$. According to the boundedness assumption on the uncertain scheduling vector $\hat{\theta}$, the error matrices would be bounded over the trajectories driven by $\theta^e = \hat{\theta} - \theta$.

A. Stability of the adaptive sliding mode observer

The design objective is to determine the discontinuous function $\nu(t)$ in (9) such that the system reaches the sliding surface $e_y(t) = 0$ and maintains a sliding motion around the surface [28].

Theorem 1: The sliding mode of the output observation error $e_y(t)$ governed by (11), which is controlled by a discontinuous signal $\nu(t)$ defined in (9)-(10), is established around $e_y(t) = 0$ if there exists a matrix K such that the following LMI condition, evaluated at each vertex $\hat{\theta}_{\mathrm{v}_i}$ for $i = 1, \ldots, M$, is satisfied,

$$C_2\left(\tilde{A}_{22}(\hat{\theta}_{\mathrm{v}_i})C_2^{-1} - K\right) + \left(\tilde{A}_{22}(\hat{\theta}_{\mathrm{v}_i})C_2^{-1} - K\right)^\top C_2^\top < 0. \tag{12}$$

Proof: We consider the following candidate Lyapunov function:

$$V_y(e_y(t), \tilde{\lambda}(t)) = \frac{1}{2}e_y^\top(t)e_y(t) + \frac{1}{2}\tilde{\lambda}(t)^\top\Lambda\tilde{\lambda}(t), \tag{13}$$

with $\Lambda = \mathrm{diag}(\bar{\lambda}_i)$, $\tilde{\lambda}(t) = \lambda(t) - \lambda^*$ and the vector λ^* to be determined. The derivative of $V_y\big(e_y(t), \tilde{\lambda}(t)\big)$ with respect to (11) is determined as

$$\begin{aligned}\dot{V}_y\big(e_y(t), \tilde{\lambda}(t)\big) =&\ \tfrac{1}{2}e_y^\top(t)\Bigg(C_2\Big(\tilde{A}_{22}(\hat{\theta})C_2^{-1} - K\Big)\\ &+ \Big(\tilde{A}_{22}(\hat{\theta})C_2^{-1} - K\Big)^\top C_2^\top\Bigg)e_y(t)\\ &+ e_y^\top(t)C_2\Big(\tilde{A}_{21}(\hat{\theta})e_1(t) + \tilde{A}^e_{21}(\hat{\theta},\theta)z_1(t)\\ &+ \tilde{A}^e_{22}(\hat{\theta},\theta)z_2(t) + \tilde{B}^e_2(\hat{\theta},\theta)u(t) - E_2w(t)\\ &- H_2f_a(t)\Big) + e_y^\top(t)\nu(t) + \dot{\lambda}^\top(t)\Lambda\tilde{\lambda}(t).\end{aligned}$$

By taking

$$\xi(\hat{\theta}) = C_2\left(\tilde{A}_{22}(\hat{\theta})C_2^{-1} - K\right) + \left(\tilde{A}_{22}(\hat{\theta})C_2^{-1} - K\right)^\top C_2^\top, \tag{14}$$

the condition $\frac{1}{2}e_y^\top(t)\xi(\hat{\theta})e_y(t) < 0$ holds if the LMI (12) is satisfied at all the vertices of $\hat{\theta}$.

Furthermore, due to the fact that $\tilde{A}_{11}(\hat{\theta})$ is stable by the design of L in (5), $e_1(t)$ is Input-to-State Stable (ISS), and hence it is bounded due to boundedness of input and output signals, as well as the error matrices $\tilde{Q}^e_{ij}$ for $i, j \in \{1, 2\}$. Therefore, if we define

$$\begin{aligned}\Psi_{\max} = \Big|C_2\Big(&\tilde{A}_{21}(\hat{\theta})e_1(t) + \tilde{A}^e_{21}(\hat{\theta},\theta)z_1(t) + \tilde{A}^e_{22}(\hat{\theta},\theta)z_2(t)\\ &+ \tilde{B}^e_2(\hat{\theta},\theta)u(t) - E_2w(t) - H_2f_a(t)\Big)\Big|_{\max},\end{aligned}$$

then it follows that

$$\begin{aligned}\dot{V}_y\big(e_y(t), \tilde{\lambda}(t)\big) \leq&\ \underbrace{\frac{1}{2}e_y^\top(t)\xi(\hat{\theta})e_y(t)}_{<0} + \big|e_y(t)\big|^\top\Psi_{\max}\\ &- |e_y(t)|^\top\lambda(t) + |e_y(t)|^\top\tilde{\lambda}(t).\end{aligned} \tag{15}$$

and hence,

$$\dot{V}_y\big(e_y(t), \tilde{\lambda}(t)\big) \leq\ \frac{1}{2}e_y^\top(t)\xi(\hat{\theta})e_y(t) + |e_y(t)|^\top\Big(\Psi_{\max} - \lambda^*\Big). \tag{16}$$

The constant vector λ^* can be selected such that $\gamma = \lambda^* - \Psi_{\max}$ is a positive vector and hence

$$\begin{aligned} \dot{V}_y\big(e_y(t), \tilde{\lambda}(t)\big) \leq\ & \frac{1}{2} e_y^\top(t) \xi(\hat{\theta}) e_y(t) - |e_y(t)|^\top \gamma \\ \leq\ & -|e_y(t)|^\top \gamma \leq 0. \end{aligned} \tag{17}$$

Consequently, $e_y(t)$ and $\tilde{\lambda}(t)$ are bounded. Moreover, by integrating (17), it follows that

$$\begin{aligned} \lim_{t\to\infty} \int_0^t |e_y(\tau)|^\top \gamma \, \mathrm{d}\tau \leq V_y\big(e_y(0), \tilde{\lambda}(0)\big) \\ - \lim_{t\to\infty} V_y\big(e_y(t), \tilde{\lambda}(t)\big) \leq \infty, \end{aligned} \tag{18}$$

since $V_y\big(e_y(t), \tilde{\lambda}(t)\big)$ is bounded. Therefore, $\lim_{t\to\infty} \int_0^t |e_y(\tau)|^\top \gamma \, \mathrm{d}\tau$ is bounded, and it follows that $\gamma^\top |e_y(t)|$ is uniformly continuous in t since $\dot{e}_y(t)$ is bounded. Thus, applying *Barbalat's Lemma* results that $\lim_{t\to\infty} \gamma^\top |e_y(t)| = 0$, which leads to $\lim_{t\to\infty} |e_y(t)| = 0$ since γ is a positive vector.

The convergence of the LPV adaptive sliding mode observer (8) is thus demonstrated, as well as the maintenance of the sliding motion around $e_y(t) = 0$, which completes the proof. ■

B. Reconstruction of the actuator fault

Once the sliding motion is maintained, the actuator fault signal $f_a(t)$ can be deduced from the principle of equivalent control which is the average effort necessary to maintain the motion on the sliding surface [28], [32]. If the induced chattering that can occur around the sliding surface is significant, the equivalent control $\nu_{\mathrm{eq}}(t)$ can be obtained from filtering the discontinuous input $\nu(t)$ in (9). In this paper, the reconstruction of the fault cannot be directly obtained from the dynamics of the output error $e_y(t) = \dot{e}_y(t) = 0$ as is the case in [27] due to the presence of unknown terms in $\dot{e}_y(t)$. Therefore, with C_2 assumed to be nonsingular, we define the reconstruction signal as [27]

$$\hat{f}_a(t) = W C_2^{-1} \nu_{\mathrm{eq}}(t), \tag{19}$$

where the matrix W has the structure of $W = \begin{bmatrix} V & H_{22}^{-1} \end{bmatrix}$ with $W H_2 = I$ for any choice of V. Substituting (19) into (11) leads to

$$\begin{aligned} -W^{-1} \hat{f}_a(t) =& \tilde{A}_{21}(\hat{\theta}) e_1(t) + \tilde{B}_2^e(\hat{\theta}, \theta) u(t) + \tilde{A}_{21}^e(\hat{\theta}, \theta) z_1(t) \\ &+ \tilde{A}_{22}^e(\hat{\theta}, \theta) z_2(t) - \tilde{E}_2 w(t) - H_2 f_a(t), \end{aligned} \tag{20}$$

and with the dynamics of $e_1(t)$, (20) yields the following new state representation

$$\begin{aligned} \dot{e}_1(t) =& A_a(\hat{\theta}) e_1(t) + B_a(\hat{\theta}, \theta) \xi_a(t, y, u), \\ f_a(t) - \hat{f}_a(t) =& C_a(\hat{\theta}) e_1(t) + D_a(\hat{\theta}, \theta) \xi_a(t, y, u), \end{aligned} \tag{21}$$

where $e_1(t)$ is the state vector and the new output is $f_a(t) - \hat{f}_a(t)$. The new system is under the augmented disturbance

$$\xi_a(t, y, u) = \Big[z_1^\top(t),\ z_2^\top(t),\ u^\top(t),\ y^\top(t),\ w^\top(t) \Big]^\top,$$

and the new state space matrices are as follows

$$\begin{aligned} & A_a(\hat{\theta}) = \tilde{A}_{11}(\hat{\theta}), \quad C_{a(\hat{\theta})} = W \tilde{A}_{21}(\hat{\theta}), \\ & B_a(\hat{\theta}, \theta) = \Big[\tilde{A}_{11}^e(\hat{\theta}, \theta),\ 0,\ \tilde{B}_1^e(\hat{\theta}, \theta),\ \tilde{A}_{12}^e(\hat{\theta}, \theta) C_2^{-1},\ -\tilde{E}_1 \Big], \\ & D_a(\hat{\theta}, \theta) = W \Big[\tilde{A}_{21}^e(\hat{\theta}, \theta),\ \tilde{A}_{22}^e(\hat{\theta}, \theta),\ \tilde{B}_2^e(\hat{\theta}, \theta),\ 0,\ -\tilde{E}_2 \Big]. \end{aligned} \tag{22}$$

The key use of the new state space representation is in the study of minimizing the effect of ξ_a on the fault reconstruction error $f_a(t) - \hat{f}_a(t)$. This way, the effect of the mismatch between the observer and the plant when $\hat{\theta} \neq \theta$ will be minimized through the components of ξ_a. In the H_∞ framework, the worst case fault estimation error energy over the input energy can be minimized as

$$\sup_{\xi_a \neq 0} \frac{\big\| f_a(t) - \hat{f}_a(t) \big\|_2}{\big\| \xi_a \big\|_2} < \eta. \tag{23}$$

Applying the Bounded Real Lemma [8] on the closed-loop state space matrix definitions (22) at all the vertices of $\hat{\theta}$ and θ, the H_∞ performance η over the trajectories of $\hat{\theta}$ and θ is guaranteed since there is a single Lyapunov function applied over the entire operating region [33].

Theorem 2: The augmented system (21) is stable and has the quadratic H_∞ performance η over the trajectories $\hat{\theta}$ and θ if there exist a matrix L of the form (5) and a symmetric $P > 0$ such that the following inequality is satisfied at each vertex $(\hat{\theta}_{\mathrm{v}_i}, \theta_{\mathrm{v}_j})$ for $i, j = 1, \ldots, M$:

$$\begin{bmatrix} A_{a_i}^\top P + P A_{a_i} & P B_{a_{i,j}} & C_{a_i}^\top \\ B_{a_{i,j}}^\top P & -\eta I & D_{a_{i,j}}^\top \\ C_{a_i} & D_{a_{i,j}} & -\eta I \end{bmatrix} < 0, \tag{24}$$

where A_{a_i}, $B_{a_{i,j}}$, C_{a_i} and $D_{a_{i,j}}$ are the augmented system matrices (22) at the vertex (i, j) for $i, j = 1, \ldots, M$.

In order to validate the proposed design approach for actuator fault reconstruction, an illustrative example is presented in the next section.

IV. Illustrative example

The effectiveness of the LPV adaptive sliding mode observer (8) is tested on a single-link flexible joint robot arm. As described in [34], the dynamical model is as follows

$$\begin{aligned} \dot{\theta}_1 =& \omega_1, \\ \dot{\omega}_1 =& \frac{1}{J_1} \big(k_1(\theta_2 - \theta_1) + k_2(\theta_2 - \theta_1)^3 \big) - \frac{B_{\mathrm{v}}}{J_1} \omega_1 + \frac{K_\tau}{J_1} u, \\ \dot{\theta}_2 =& \omega_2, \\ \dot{\omega}_2 =& -\frac{1}{J_2} \big(k_1(\theta_2 - \theta_1) + k_2(\theta_2 - \theta_1)^3 \big) - \frac{m_l g h}{J_2} \sin\theta_2 \\ & + \Phi(\theta_1, \omega_1, \theta_2, \omega_2, t), \end{aligned} \tag{25}$$

where θ_1 and θ_2 are the motor and link positions, respectively; ω_1 and ω_2 are the velocities, J_1 is the inertia of the DC motor, J_2 is the inertia of the link, $2h$ is the length of the link and m_l represents its mass, B_v is the viscous friction, k_1 and k_2 are both positive constants and K_τ is the amplifier gain. The motor position, motor velocity, and the sum of link velocity and link position are assumed to be measured. The uncertainty $\Phi(\theta_1, \omega_1, \theta_2, \omega_2, t)$ affecting the system satisfies $|\Phi(\cdot)| \leq |\omega_1 \sin \omega_2| \exp(-t)$. The scheduling vector ρ including all the nonlinear terms in (25) is defined as

$$\begin{aligned} \rho_1 &= \theta_1^2 + 2\theta_2, \\ \rho_2 &= \theta_2^2, \\ \rho_3 &= \frac{\sin \theta_2}{\theta_2}, \end{aligned} \tag{26}$$

and the matrices of the LPV model representation (1) are thus written as

$$A(\rho) = \begin{bmatrix} 0 & 1 & 0 & 0 \\ -\frac{k_1}{J_1} - \frac{k_2}{J_1}\rho_1 & -\frac{B_v}{J_1} & \frac{k_1}{J_1} + \frac{k_2}{J_1}\rho_2 & 0 \\ 0 & 0 & 0 & 1 \\ \frac{k_1}{J_2} + \frac{k_2}{J_2}\rho_1 & 0 & -\frac{k_1}{J_2} - \frac{k_2}{J_2}\rho_2 - \frac{m_l g h}{J_2}\rho_3 & 0 \end{bmatrix},$$

$$B = \begin{bmatrix} 0 \\ \frac{K_\tau}{J_1} \\ 0 \\ 0 \end{bmatrix}, C = \begin{bmatrix} 1\,0\,0\,0 \\ 0\,1\,0\,0 \\ 0\,0\,1\,1 \end{bmatrix}, E = \begin{bmatrix} 0 & 0 & 0 & 1 \end{bmatrix}^\top, H = B. \tag{27}$$

In order to obtain the LPV model (27) of the form (3) as in Assumption 3, a change of coordinate is necessary with the matrix

$$\mathcal{T} = \begin{bmatrix} 0 & 0 & 1 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & -1 & -1 \end{bmatrix},$$

and the resulting matrices are of the form

$$\begin{aligned} &A_{11} = -1,\ B_2 = H_2 = \begin{bmatrix} 0 \\ -\frac{K_\tau}{J_1} \\ 0 \end{bmatrix},\ E_2 = \begin{bmatrix} 0 \\ 0 \\ -1 \end{bmatrix}, \\ &C_2 = \begin{bmatrix} 1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & -1 \end{bmatrix}. \end{aligned} \tag{28}$$

The T transformation in (4), introduced in Assumption 4, is not needed here since $e_1(t)$ is stable due to the fact that $A_{11} < 0$. Random faults are introduced in the measured scheduling variables reaching up to 50% of their real values as seen in Figure 1. The bounds of $\hat{\rho}$ are determined according to the maximum values of faults. The LMI (12) is solved for the 8 vertices based on the bounds of $\hat{\rho}$ and the observer gain is hence obtained as

$$K = 10^3 \begin{bmatrix} 8.0134 & -0.0265 & 0.0118 \\ 0.0212 & -8.0253 & -0.4933 \\ -0.0077 & 0.547 & -8.044 \end{bmatrix}.$$

Finally, for the reconstruction of the actuator fault, a suitable choice of the decoupling matrix for (20) is $W = \begin{bmatrix} 1 & -0.0463 & 0 \end{bmatrix}$. The H_∞ performance over the trajectories of $\hat{\theta}$ and θ, using the 64 vertices for solving the LMI (24), is verified with a minimal value of $\eta = 0.01127$.

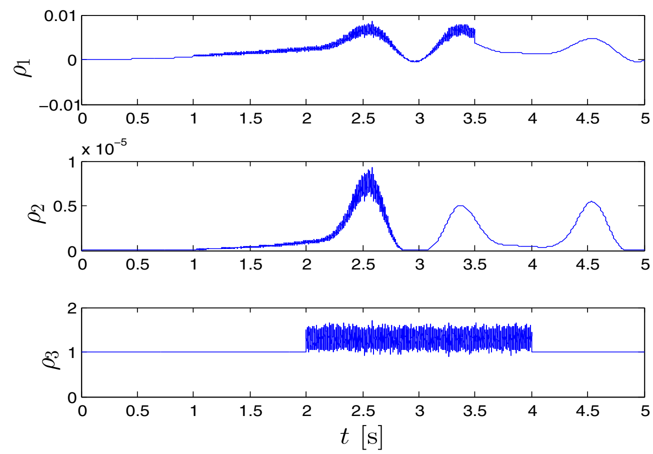

Fig. 1. Faulty measured scheduling variables.

For the simulation study, a linear state feedback $u = \begin{bmatrix} -16.2 & -12.1 & -39.7 & -25.6 \end{bmatrix} x$ is used to stabilize the system. The actuator fault starting with a ramp then becoming sinusoidal is considered for this example. As shown in Figure 2, the state estimation errors converge to zero using the proposed LPV adaptive siding mode observer (8). The fault is very well reconstructed as shown in Figure 3. The detection of the fault presence is instantaneous (in 0.02s) and the fault is estimated with a mean square percentage error of 0.032% of the real fault signal.

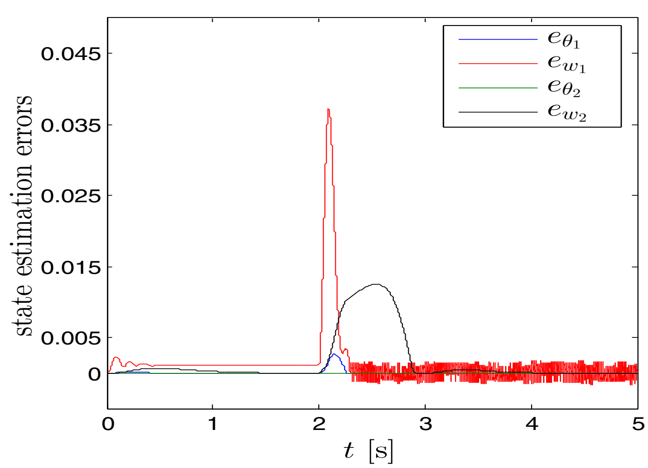

Fig. 2. State estimation errors with the LPV adaptive sliding mode observer.

V. Conclusions

In this paper, an LPV adaptive sliding mode observer is developed for actuator fault diagnosis and reconstruction of a class of LPV systems with uncertain scheduling variables. By means of an adaptive gain, the proposed observer is able

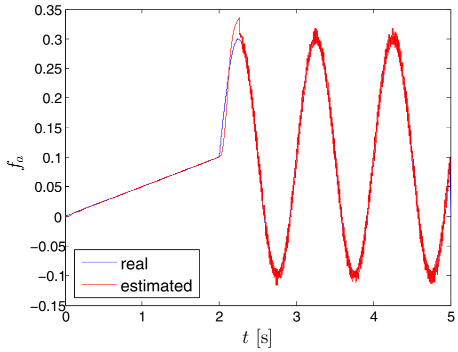

Fig. 3. Fault reconstruction with the LPV adaptive sliding mode observer.

to cope with faults and uncertainties in the LPV systems. The actuator faults can be estimated after minimizing the effect of the uncertainties and imperfect scheduling variables on the fault reconstruction. The LPV sliding mode observer tested in Matlab/Simulink reveals the simultaneous detection and the fast tracking of the fault profile without the need for any filtering technique or a priori knowledge on the fault upper bound.

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