2017 American Control Conference
Sheraton Seattle Hotel
May 24–26, 2017, Seattle, USA

Coverage Control of Moving Sensor Networks with Multiple Regions of Interest*

Farshid Abbasi†, Afshin Mesbahi† and Javad Mohammadpour Velni†

Abstract— **This paper addresses the coverage control problem in environments where several regions of interest exist. To this purpose, a heterogeneous group of robots are deployed to minimize a cost function defined with respect to various spatial probability density functions, each of which describes a desired area for a different group of robots. Each region of interest is assigned to a group of robots with respect to their dynamics and sensing capabilities. A distributed coverage scheme is proposed to allow adjusting to the environment with several important areas in a collaborative way. The regions with higher importance would be covered with an appropriate number of robots. The proposed method also allows for a better allocation of robots to guarantee the desired coverage over the region. Two numerical examples are finally given to examine the proposed coverage approach in case of multiple regions of interest that may need to be covered by a certain number of robots.**

I. Introduction

Analytical methods have been developed for deployment of a group of robots to accomplish assigned tasks, such as coverage, in an environment known *a priori*. Typical applications of the coverage problem include search, surveillance, target detection and rescue operations, sensing, and data collection [1], [2], [3], [4]. The major question in coverage problem is how to share the workload in a reliable and efficient way while performing in a distributed way. The distributed method proposed in [5] finds a locally optimal coverage in an environment with a homogeneous group of robots based on the Voronoi diagram framework, where each robot implements a control law designed based on the gradient descent method that minimizes the coverage cost in space and time leading to an optimal partitioning.

A multi-robot system can generally be considered as a homogeneous or heterogeneous system deployed to perform assigned tasks [6], [7], [8]. The generalized Voronoi diagrams depending on a set of weights are used to handle the heterogeneous groups of robots as presented in [9], [10], [11]. Furthermore, due to the need for collecting various types of data from a given region, robots may be equipped with different types of sensors to measure the environmental parameters with different rates and precision [12], [13], [14], [8]. It is also possible that a sensor installed on a robot has a particular operational characteristics, and hence, it may be more suitable to be deployed in a certain part of the region. The present work considers the case, where there are more than one important region in the environment, where each important region is assigned to a set of robots that will cover the area using their equipped sensors.

Existing coverage control approaches plan a local optimal path for each robot to address the coverage problem neglecting the possible differences in the dynamics of the robots [15]. For instance, a heterogeneous group of unmanned aerial vehicles (UAVs) and unmanned ground vehicles (UGVs) were deployed in [16], [17] to cover large areas, where it was shown that the dynamical differences may lead to several issues. For example, the UGVs are not able to move as fast as UAVs, but they can be equipped with highly accurate sensors [15], [17]. Hence, it is realistic in many practical applications to take into account the dynamics of the robots in planning the optimal path. This paper also attempts to present a distributed approach that enables the robots to be deployed with respect to their dynamics. The agents with similar dynamics can be assigned to the same region to accomplish the coverage task in a more efficient way. A long-distance important region, with respect to the initial positions of robots, is assigned to a group of robots that have the dynamic capabilities to move faster than others. For instance, in the UAV/UGV example given above, the UAVs are desired to cover the regions far from the locations they are initially deployed while the UGVs pursue tasks close to their initial deployed position.

A team-based coverage control scheme has been introduced in [18] to facilitate the deployment of a heterogeneous group of robots. The proposed algorithm uses heterogeneous teams of robots to handle the coverage of the given environment, but the number of teams and team members are assumed to be independent of the environment. The present work introduces a new coverage control scheme that can allow deploying groups of heterogeneous robots in order to handle the coverage problem in an environment that consists of multiple important regions. Each important region is assigned to a group of robots by taking into account their dynamics and sensing capabilities. To this aim, the density function associated with each robot may differ from its neighbors to reflect the difference in their associated regions of interest. Each robot might team up with others based on its density function, associated dynamics, or sensing characteristics. Therefore, the proposed coverage strategy would make it possible to improve reliability, accuracy, and flexibility of the deployment algorithm by taking into account the differences in the embedded sensors and dynamics of robots through offering an alternative solution method for the underlying sensing cost function. In the proposed formulation, the importance functions associated with the

*This research was supported in part by a grant from the University of Georgia Research Foundation, Inc.

† College of Engineering, The University of Georgia, Athens, GA 30602, E-mail: javadm@uga.edu

neighboring agents may differ form each other. This implies that, unlike the previous coverage control algorithms, there will be an additional term in the control law that takes into account the potential difference in the importance function of the robots sharing boundaries. Each robot can calculate its Voronoi cell by knowing not only the position of the neighboring robots, but also a data set associated with the difference in their density functions. The required information for calculating the Voronoi cells is obtained through the adjust-communication radius algorithm first developed in [5].

The remainder of this paper is structured as follows. Definitions and the problem statement are provided in Section II. Section III introduces a new optimization problem suited for the coverage problem in the presence of several regions of interest. The asymptotic convergence of the group of agents imposing the proposed control law is proven through the use of the Barbalat's lemma. Section IV presents simulation results to illustrate the proposed solution method for the coverage problem in the environment with multiple regions of interest with different degrees of importance.

Notations

We use $\mathbb{N}$, $\mathbb{R}$, and $\mathbb{R}_+$ to denote the sets of natural, real, and nonnegative real numbers. Also, I_r denotes $r \times r$ identity matrix. We define $\mathcal{Q}$ as a convex polytope in $\mathbb{R}^2$ and let $\mathscr{Q} = \{\mathcal{Q}_1, \mathcal{Q}_2, \dots, \mathcal{Q}_N\}$ be a *partition* of $\mathcal{Q}$ as a collection of closed subsets with disjoint interiors. The boundary of $\mathcal{Q}$ is shown by $\mathcal{Q}^B$. The shared edge of the Voronoi cell V_m with Voronoi cell V_s is shown by V_{ms}^B that is $V_m^B \cap V_s^B$. Moreover, the so-called *distribution density function* (importance function) is denoted by φ, where $\varphi : \mathcal{Q} \to \mathbb{R}_+$ represents the probability of some phenomenon occurring over space $\mathcal{Q}$. The function φ is assumed to be measurable and absolutely continuous. The Euclidean distance function is denoted by $\|\cdot\|$ and $|\mathcal{Q}|$ represents the Lebesgue measure of convex subset $\mathcal{Q}$. With p_i defined as the location of the i^{th} agent, the vector set $\mathcal{P} = (p_1, p_2, \dots, p_N)$ denotes the location of N agents in the space $\mathcal{Q}$.

II. Preliminaries and Problem Statement

The objective of the coverage related tasks is to ensure a high sensing performance for a mobile network of sensors. This is formulated as an optimization problem that aggregates the sensing performance of all the robots into a so-called sensing cost function. This section gives an overview of the sensing cost function and the necessary modifications to cope with the complexity of the problem at hand.

A. Voronoi partitions

This paper aims at addressing the problems of agents deployment and partitioning to efficiently handle multiple tasks. The main objective of this work is to adopt the multi tasking concept in the agents deployment problem and partitioning framework. To do so, we first need to define an optimization problem that can handle the partitioning and deployment within the defined polytope $\mathcal{Q}$. The optimization problem should consider various importance functions representing the difference in the received sensory data or the assigned task in the context of coverage control.

We first partition the polytope $\mathcal{Q}$ into a set of Voronoi cells $\mathcal{V}(\mathcal{P}) = \{V_1, V_2, \dots, V_N\}$ that are known to provide the optimal partitioning for a set of agents with fixed locations at a given space [5] as

$$V_m = \{q \in \mathcal{Q} | \, \|q - p_m\| \leq \|q - p_r\|, r = 1, \dots, N, r \neq m\}, \tag{1}$$

where p_m denotes the location of m^{th} agent in $\mathcal{Q}$ and $m \in \{1, \dots, N\}$. The importance function associated with one agent can differ from another; such a difference can well represent heterogeneity in their embedded sensing devices or assigned tasks in the distributed system of agents. We recall the basic characteristics of the Voronoi partitions including their associated mass, centroid, and polar moment of inertia defined as in [5].

III. Coverage Control in an Environment with Multiple Regions of Interest

Due to the possible difference in the agents sensing capabilities, it is likely that the agents can carry different sensors. This is addressed in the present work by introducing an alternative formulation of the coverage problem for heterogeneous agents pursuing various tasks in a region characterized by different importance functions. The deployment task of interest in this paper can be addressed by solving an optimization problem that represents the agents with various regions of interest collectively. The optimization problem seeks to achieve the optimal deployment over the convex polytope $\mathcal{Q}$, where each agent's region needs to be determined. Hence, the following cost function is defined

$$\mathcal{G}(\mathcal{P}, \mathscr{Q}) = \sum_{m=1}^{N} \int_{\mathcal{Q}_m} \|q - p_m\|^2 \varphi_m(q) \mathrm{d}q, \tag{2}$$

in which the sensing performance is considered as $f(\|q - p_m\|) = \|q - p_m\|^2$ for the m^{th} agent with the importance function of φ_m. The solution to minimizing $\mathcal{G}$ in (2) gives a local minimum to the deployment problem, where agents are collaborating and heterogeneous. It can be shown that among different partitioning schemes, the Voronoi partitions are optimum in the sense of minimizing defined cost function (2) [5]. Hence, for a given set of agents position $\mathcal{P} \in \mathcal{Q}$ and a *partition* $\mathscr{Q}$ of $\mathcal{Q}$, it satisfies

$$\mathcal{G}(\mathcal{P}, \mathcal{V}(\mathcal{P})) \leq \mathcal{G}(\mathcal{P}, \mathscr{Q}), \tag{3}$$

which implies that the Voronoi cells represent the optimum partitioning. To obtain the optimum configuration of the agents, we need to solve the problem of minimizing (2). In order to avoid the complexity involved in obtaining the global minimum of an NP-hard (Non-deterministic Polynomial-time hard) problem, the local minimum of the cost function is sought for by taking the derivative of the sensing cost function with respect to the agents position as

$$\frac{\partial \mathcal{G}}{\partial p_m} = \frac{\partial}{\partial p_m} \sum_{r=1}^{N} \int_{V_r} \|q - p_r\|^2 \varphi_r(q) \mathrm{d}q, \quad m = 1, \dots, N. \tag{4}$$

The solution to this problem differs from the conventional sensing cost functions due to various importance functions involved that represent heterogeneity. The derivative with respect to the coordinates of agent p_m is obtained as

$$\frac{\partial \mathcal{G}}{\partial p_m} = \int_{V_m} \frac{\partial}{\partial p_m} \|q - p_m\|^2 \varphi_m(q) \mathrm{d}q + \sum_{r=1}^{N} \int_{V_r^B} \|q - p_r\|^2 \varphi_r(q) \frac{\partial V_r^B}{\partial p_m} \mathcal{N}_r \mathrm{d}q, \quad (5)$$

where V_r^B represents the boundary of the Voronoi cell V_r. As it can be inferred from the definition of the Voronoi partitioning, the boundary of the Voronoi cell V_r that is in the neighborhood of m^{th} robot is dependent on p_m.

Remark 1: The agents whose Voronoi cells do not share any edges with the Voronoi cell associated with p_m are independent of p_m. This implies that $\frac{\partial V_r^B}{\partial p_m} = 0$ for any $r = 1, \dots, N$ that $p_r \notin \mathcal{N}_{p_m}$ where $\mathcal{N}_{p_m}$ represents the set of agents that share boundaries with m^{th} agent.

Remark 2: The integral on each boundary shared with neighboring agents is the same for agents on both sides except that the normals have opposite signs, i.e., $\mathcal{N}_{sm} = -\mathcal{N}_{ms}$, where $\mathcal{N}_{ms}$ is the normal vector for the edge of the Voronoi cell V_m, i.e., V_{ms}^B, that is shared with another Voronoi cell V_s.

The last term in (5) can be rewritten as follows

$$\sum_{r=1}^{N} \int_{V_r^B} \|q - p_r\|^2 \varphi_r(q) \frac{\partial V_r^B}{\partial p_m} \mathcal{N}_r \mathrm{d}q = \sum_{p_r \in \mathcal{N}_{p_m}} \int_{V_r^B} \|q - p_r\|^2 \varphi_r(q) \frac{\partial V_r^B}{\partial p_m} \mathcal{N}_r \mathrm{d}q + \sum_{p_r \notin \mathcal{N}_{p_m}} \int_{V_r^B} \|q - p_r\|^2 \varphi_r(q) \frac{\partial V_r^B}{\partial p_m} \mathcal{N}_r \mathrm{d}q. \quad (6)$$

According to Remark 1, the last term is reduced to the integral on the boundary of the agent m^{th} agent. Then, we obtain the following

$$\begin{aligned}\sum_{r=1}^{N} \int_{V_r^B} \|q - p_r\|^2 \varphi_r(q) \frac{\partial V_r^B}{\partial p_m} \mathcal{N}_r \mathrm{d}q &= \sum_{p_r \in \mathcal{N}_{p_m}} \int_{V_r^B} \|q - p_r\|^2 \varphi_r(q) \frac{\partial V_r^B}{\partial p_m} \mathcal{N}_r \mathrm{d}q + \int_{V_m^B} \|q - p_m\|^2 \varphi_m(q) \frac{\partial V_m^B}{\partial p_m} \mathcal{N}_m \mathrm{d}q \\ &= \sum_{p_r \in \mathcal{N}_{p_m}} \int_{V_{rm}^B} \|q - p_r\|^2 \varphi_r(q) \frac{\partial V_{rm}^B}{\partial p_m} \mathcal{N}_{rm} \mathrm{d}q + \sum_{p_r \in \mathcal{N}_{p_m}} \int_{V_{mr}^B} \|q - p_m\|^2 \varphi_m(q) \frac{\partial V_{mr}^B}{\partial p_m} \mathcal{N}_{mr} \mathrm{d}q. \quad (7)\end{aligned}$$

Using Remark 2 and the following equality

$$\frac{\partial V_{mr}^B}{\partial p_m} = \frac{\partial V_{rm}^B}{\partial p_m}, \quad (8)$$

it can be concluded that

$$\sum_{r=1}^{N} \int_{V_r^B} \|q - p_r\|^2 \varphi_r(q) \frac{\partial V_r^B}{\partial p_m} \mathcal{N}_r \mathrm{d}q = \sum_{p_r \in \mathcal{N}_{p_m}} \int_{V_{mr}^B} (\|q - p_m\|^2 \varphi_m(q) - \|q - p_r\|^2 \varphi_r(q)) \frac{\partial V_{mr}^B}{\partial p_m} \mathcal{N}_{mr} \mathrm{d}q. \quad (9)$$

From the definition of the Voronoi cell (1), we have $\|q - p_m\| = \|q - p_r\|$ for any $r \in \mathcal{N}_{p_m}$. Hence, the following is obtained

$$\sum_{r=1}^{N} \int_{V_r^B} \|q - p_r\|^2 \varphi_r(q) \frac{\partial V_r^B}{\partial p_m} \mathcal{N}_r \mathrm{d}q = \sum_{p_r \in \mathcal{N}_{p_m}} \int_{V_{mr}^B} \|q - p_m\|^2 \big(\varphi_m(q) - \varphi_r(q)\big) \frac{\partial V_{mr}^B}{\partial p_m} \mathcal{N}_{mr} \mathrm{d}q. \quad (10)$$

It is noted that each agent might also be the neighbor of other agents that pursue a similar task through the same importance function. If that is the case, then the result of (10) is equal to zero. However, this is not the case in general and hence, we need to evaluate the integral on the given boundaries.

The line to which the points on the agents boundaries belong can be described by

$$(\mathcal{N}_{mr})^\top (q - \frac{p_r + p_m}{2}) = 0, \quad q \in V_{mr}^B, \quad (11)$$

where V_{mr}^B is the shared boundary between agents m and r. The normal vector $\mathcal{N}_{mr}$ associated with V_{mr}^B is obtained by

$$\mathcal{N}_{mr} = \frac{p_r - p_m}{\|p_r - p_m\|}. \quad (12)$$

The partial derivative of (11) with respect to p_m is obtained as follows

$$\frac{\partial \mathcal{N}_{mr}}{\partial p_m} (q - \frac{p_r + p_m}{2}) + (\frac{\partial V_{mr}^B}{\partial p_m} - \frac{1}{2}) \mathcal{N}_{mr} = 0, \quad (13)$$

where

$$\frac{\partial \mathcal{N}_{mr}}{\partial p_m} = \frac{\mathcal{N}_{mr} (\mathcal{N}_{mr})^\top - I_2}{\|p_r - p_m\|}. \quad (14)$$

By substituting (14) into (13), we obtain

$$\frac{\partial V_{mr}^B}{\partial p_m} \mathcal{N}_{mr} = \frac{\mathcal{N}_{mr} (\mathcal{N}_{mr})^\top - I_2}{\|p_r - p_m\|} (\frac{p_r + p_m}{2} - q) + \frac{1}{2} \mathcal{N}_{mr}, \quad q \in V_{mr}^B. \quad (15)$$

Substituting (15) back into (10) leads to

$$\sum_{r=1}^{N}\int_{V_r^B}\|q-p_r\|^2\varphi_r(q)\frac{\partial V_r^B}{\partial p_m}\mathcal{N}_r\mathrm{d}q = \sum_{p_r\in\mathcal{N}_{p_m}}\frac{\mathcal{N}_{mr}(\mathcal{N}_{mr})^\top - I_2}{\|p_r-p_m\|}\int_{V_{mr}^B}\Big(\|q-p_m\|^2(\varphi_m(q)-\varphi_r(q))\frac{p_r+p_m}{2} - \|q-p_m\|^2\big(\varphi_m(q)-\varphi_r(q)\big)q\Big)\mathrm{d}q + \frac{1}{2}\mathcal{N}_{mr}\int_{V_{mr}^B}\|q-p_m\|^2\big(\varphi_m(q)-\varphi_r(q)\big)\mathrm{d}q. \quad (16)$$

By rearranging the terms in (16), we obtain

$$\sum_{r=1}^{N}\int_{V_r^B}\|q-p_r\|^2\varphi_r(q)\frac{\partial V_r^B}{\partial p_m}\mathcal{N}_r\mathrm{d}q = \sum_{p_r\in\mathcal{N}_{p_m}}\Big(\frac{\mathcal{N}_{mr}(\mathcal{N}_{mr})^\top - I_2}{\|p_r-p_m\|}\frac{p_r+p_m}{2}+\frac{1}{2}\mathcal{N}_{mr}\Big)\int_{V_{mr}^B}\|q-p_m\|^2\big(\varphi_m(q)-\varphi_r(q)\big)\mathrm{d}q - \frac{\mathcal{N}_{mr}(\mathcal{N}_{mr})^\top - I_2}{\|p_r-p_m\|}\int_{V_{mr}^B}\|q-p_m\|^2\big(\varphi_m(q)-\varphi_r(q)\big)q\mathrm{d}q. \quad (17)$$

The derivative (5) can then be rewritten as

$$\frac{\partial\mathcal{G}}{\partial p_m} = \int_{V_m}\frac{\partial}{\partial p_m}\|q-p_m\|^2\varphi_m(q)\mathrm{d}q + \sum_{p_r\in\mathcal{N}_{p_m}}\Big(\frac{\mathcal{N}_{mr}(\mathcal{N}_{mr})^\top - I_2}{\|p_r-p_m\|}\frac{p_r+p_m}{2}+\frac{1}{2}\mathcal{N}_{mr}\Big)\int_{V_{mr}^B}\|q-p_m\|^2\big(\varphi_m(q)-\varphi_r(q)\big)\mathrm{d}q - \frac{\mathcal{N}_{mr}(\mathcal{N}_{mr})^\top - I_2}{\|p_r-p_m\|}\int_{V_{mr}^B}\|q-p_m\|^2\big(\varphi_m(q)-\varphi_r(q)\big)q\mathrm{d}q. \quad (18)$$

Further simplification of (18) using the equations of mass and centroid results in

$$\frac{\partial\mathcal{G}}{\partial p_m} = -2M_{V_m}(C_{V_m}-p_m) + \sum_{p_r\in\mathcal{N}_{p_m}}\Big(\frac{\mathcal{N}_{mr}(\mathcal{N}_{mr})^\top - I_2}{\|p_r-p_m\|}\frac{p_r+p_m}{2}+\frac{1}{2}\mathcal{N}_{mr}\Big)\int_{V_{mr}^B}\|q-p_m\|^2\big(\varphi_m(q)-\varphi_r(q)\big)\mathrm{d}q - \frac{\mathcal{N}_{mr}(\mathcal{N}_{mr})^\top - I_2}{\|p_r-p_m\|}\int_{V_{mr}^B}\|q-p_m\|^2\big(\varphi_m(q)-\varphi_r(q)\big)q\mathrm{d}q. \quad (19)$$

Considering the following notations

$$\mathcal{G}_{mr} = \int_{V_{mr}^B}\|q-p_m\|^2\big(\varphi_m(q)-\varphi_r(q)\big)\mathrm{d}q, \quad (20)$$

$$\mathcal{L}_{mr} = \int_{V_{mr}^B}\|q-p_m\|^2(\varphi_m(q)-\varphi_r(q))q\mathrm{d}q, \quad (21)$$

equation (19) can be written as

$$\frac{\partial\mathcal{G}}{\partial p_m} = -2M_{V_m}(C_{V_m}-p_m) + \sum_{p_r\in\mathcal{N}_{p_m}}(\frac{\mathcal{N}_{mr}(\mathcal{N}_{mr})^\top - I_2}{\|p_r-p_m\|}\frac{p_r+p_m}{2}+\frac{1}{2}\mathcal{N}_{mr})\mathcal{G}_{mr} - (\frac{\mathcal{N}_{mr}(\mathcal{N}_{mr})^\top - I_2}{\|p_r-p_m\|})\mathcal{L}_{mr}. \quad (22)$$

As observed from (22), we need to obtain the integral on the boundaries of the agents' Voronoi for $\mathcal{G}_{mr}$ and $\mathcal{L}_{mr}$ that take into account the effect of the difference in the importance function of the neighboring agents.

A. Controller design

The gradient-decent based control law is utilized here to guarantee the convergence of the agents to their equilibrium point while pursing various objectives. The following dynamics is imposed on each agent

$$\dot{p}_m = u_m = \frac{K_m}{2M_{V_m}}\Big(-\frac{\partial\mathcal{G}}{\partial p_m}\Big) = \frac{K_m}{2M_{V_m}}\Big(2M_{V_m}(C_{V_m}-p_m)-\gamma_m\Big), \quad m=1,\ldots,N, \quad (23)$$

where K_m is a positive scalar and

$$\gamma_m = \sum_{p_r\in\mathcal{N}_{p_m}}(\frac{\mathcal{N}_{mr}(\mathcal{N}_{mr})^\top - I_2}{\|p_r-p_m\|}\frac{p_r+p_m}{2}+\frac{1}{2}\mathcal{N}_{mr})\mathcal{G}_{mr} - (\frac{\mathcal{N}_{mr}(\mathcal{N}_{mr})^\top - I_2}{\|p_r-p_m\|})\mathcal{L}_{mr}. \quad (24)$$

By dividing K_m by varying $2M_{V_m}$, the control input is normalized to distribute the effect of both terms in the controller design. As expected, the first term drives the agent towards its centroid while the second term is associated with various regions of interest when multiple agents collaborate. In other words, the above control law ensures that the agents move to their optimum location while taking into account the difference in importance functions.

B. Computation of the Voronoi cells

The Voronoi cells associated with each agent require a set of information to be computed over time. Based on the algorithm presented in [5], the agents are able to compute their Voronoi cells by communicating with the neighboring agents. In order to obtain the control law for each agent in the proposed approach of this paper, the agents need to receive the integral of $\|q-p_r\|\varphi_r(q)$ and $\|q-p_r\|\varphi_r(q)q$ on the edge shared with other agents from the neighboring agents. Even though this increases the amount of data to be exchanged through the communication link, however, it eliminates the need for equipping all the agents with the same sensing devices in practical applications. In fact, the proposed distributed approach enables a group of heterogeneous agents capable of accomplishing the coverage task while acquiring certain information from their neighbors.

C. *Convergence of the proposed controller*

The proposed controller drives the agents to their centroid while taking the heterogeneity of the agents into account. To ensure the convergence of the agents to their collective local optimum, the following lemma is proposed.

Lemma 3.1: The agents converge to a local minimum by imposing the control law (23). That is,

$$\lim_{\tau\to\infty}\Big\| -2M_{V_m}(\tau)(C_{V_m}(\tau)-p_m(\tau))+\gamma_m(\tau)\Big\|=0, \tag{25}$$

for $\forall m \in \{1,\ldots,N\}$.

Proof: The asymptotic behavior cannot be proved through invoking the invariant set theorem for time-varying systems; we hence use the Barbalat's lemma to prove the asymptotic convergence of the group of agents when each agent follows an optimal configuration. To this aim, the Lyapunov-like function associated with each team is defined as

$$V=\mathcal{G}(\cdot,\cdot). \tag{26}$$

The derivative of this function is obtained as

$$\dot{V}=\sum_{m=1}^{N}(\frac{\partial p_m}{\partial \tau})^{\top}\frac{\partial \mathcal{G}}{\partial p_m}. \tag{27}$$

Substituting (19) and (23) into (27), we obtain

$$\dot{V}=-\sum_{m=1}^{N}\frac{K_m}{2M_{V_m}}\Big(-2M_{V_m}(C_{V_m}-p_m)+\gamma_m\Big)^{\top}\Big(-2M_{V_m}(C_{V_m}-p_m)+\gamma_m\Big). \tag{28}$$

Since M_{V_m} and K_m are positive scalars, it can be concluded that the derivative (28) is non-positive, $\dot{V}\leq 0$. Due to the positivity of the cost function $\mathcal{G}$, it is also concluded that the Lyapunov-like function (26) is non increasing and hence lower bounded. As shown in [19], $\ddot{V}(\tau)$ is uniformly bounded that results in uniform continuity of the $\dot{V}(\tau)$. Now, due to the boundedness of $V(\tau)$ and the continuity of $\dot{V}(\tau)$, it is concluded by Barbalat's lemma that

$$\lim_{\tau\to\infty}\dot{V}(\tau)=0 \quad\Longrightarrow\quad \lim_{\tau\to\infty}\| -2M_{V_m}(\tau)(C_{V_m}(\tau)-p_m(\tau))+\gamma_m(\tau)\|=0, \tag{29}$$

for $\forall m \in \{1,\ldots,N\}$. ■

IV. Simulation Results and Discussion

Simulation results are shown here to examine the efficiency of the proposed coverage control approach. The objective is to determine the coverage configuration and the path taken by each robot to yield the optimal deployment. In the first scenario, three different importance functions are assigned to three groups, each composed of six robots. As an initial configuration, the robots are deployed randomly to accomplish their associated coverage task while

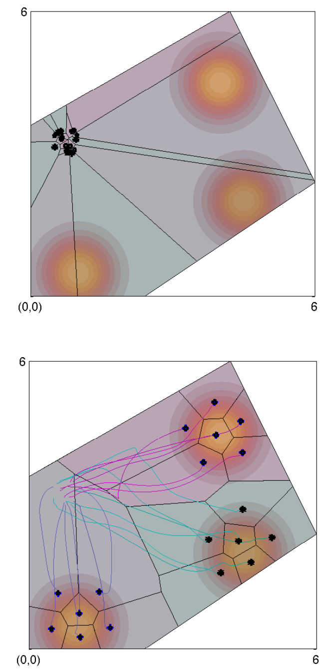

Fig. 1. Initial (top) and final configurations (bottom) for three groups of six agents, each of which pursues a different coverage task.

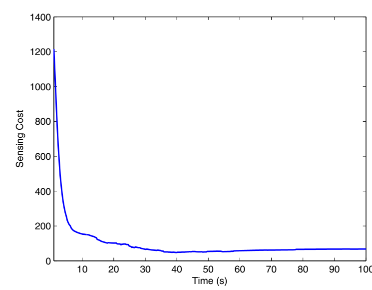

Fig. 2. Convergence of the sensing cost for the first scenario.

communicating with agents pursuing a different task. This illustrates the heterogeneity of the agents in terms of the underlying coverage density function that represents the potential differences in the dynamics or sensing capabilities of the robots. As shown in Figure 1, the robots collaborate to ultimately converge to their assigned region of interest. It is noted that the controller gain is chosen to be $K_m = 0.073$ for the first example. Furthermore, the convergence of the sensing cost function is shown in Figure 2. As the second scenario, three groups composed of 8, 4, and 6 robots are deployed to cover a given region. The goal here is to demonstrate how the proposed coverage control method works for there multiple regions of interest with various degrees of importance (various values of the variance). Hence, the proposed method enables us to deploy different number of robots to each region with respect to the complexity of the underlying sensing or coverage tasks. The control gain for this scenario is chosen to be $K_m = 0.012$. It is also noted that throughout this work, it is assumed that the importance functions are known *a priori*. As the simulation results show,

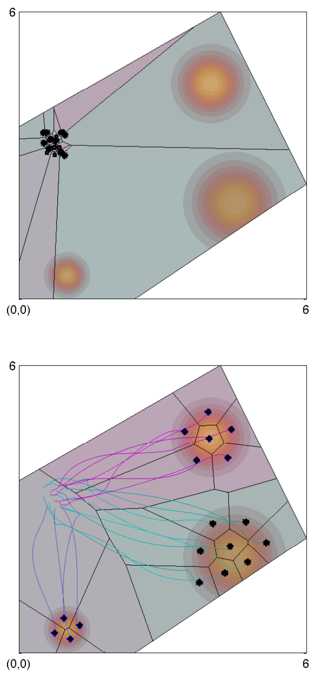

Fig. 3. Initial (top) and final configurations (bottom) for three groups of 8, 4, and 6 agents, each of which is deployed to cover a region of interest. The results show the flexibility of the proposed approach to cope with problems, where there is a need to assign various number of robots to different regions.

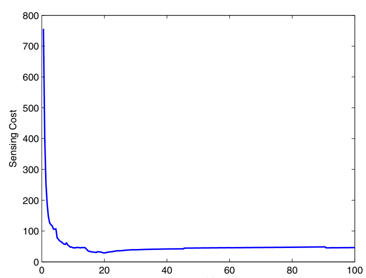

Fig. 4. Convergence of the sensing cost for the second scenario.

the ultimate goal is achieved, where the larger regions are assigned to a higher number of agents or the agents with better sensing performance are given a larger region to cover. The convergence of the sensing cost function for the second scenario is shown in Figure 4.

V. Concluding Remarks

In this paper, a coverage control approach is proposed to cover environments with several important regions. The proposed method enables the deployment of a group of heterogeneous robots with potentially different dynamics and sensing capabilities to perform a desired task while minimizing the sensing cost. Each region of interest is allocated to a group of agents based on the similarity in sensing capabilities and dynamics. Also, the number of robots that need to be deployed over the regions of interest is decided based on the number of importance functions and their associated degree of importance. Robots can compute their Voronoi cells relying on a distributed communication algorithm with their neighbor robots. The proposed approach is an attempt to modify and improve existing methods in a way that several groups of heterogeneous robots can divide the region among themselves based on their capabilities.

References

[1] Y. Liu and G. Nejat, "Robotic urban search and rescue: A survey from the control perspective," *Journal of Intelligent and Robotic Systems*, vol. 72, no. 2, pp. 147–165, 2013.

[2] S. Li, R. Kong, and Y. Guo, "Cooperative distributed source seeking by multiple robots: Algorithms and experiments," *IEEE/ASME Transactions on Mechatronics*, vol. 19, no. 6, pp. 1810–1820, Dec 2014.

[3] A. Wallar, E. Plaku, and D. Sofge, "Reactive motion planning for unmanned aerial surveillance of risk-sensitive areas," *Automation Science and Engineering, IEEE Transactions on*, vol. 12, no. 3, pp. 969–980, July 2015.

[4] S. Lee, Y. Diaz-Mercado, and M. Egerstedt, "Multirobot control using time-varying density functions," *Robotics, IEEE Transactions on*, vol. 31, no. 2, pp. 489–493, April 2015.

[5] J. Cortes, S. Martinez, T. Karatas, and F. Bullo, "Coverage control for mobile sensing networks," *Robotics and Automation, IEEE Transactions on*, vol. 20, no. 2, pp. 243–255, April 2004.

[6] E. Bakolas, "Partitioning algorithms for homogeneous multi-vehicle systems with planar rigid body dynamics," in *Decision and Control (CDC), 2014 IEEE 53rd Annual Conference on*, Dec 2014, pp. 5393–5398.

[7] O. Baron, O. Berman, D. Krass, and Q. Wang, "The equitable location problem on the plane," *European Journal of Operational Research*, vol. 183, no. 2, pp. 578–590, 2007.

[8] H. Mahboubi, K. Moezzi, A. G. Aghdam, and K. Sayrafian-Pour, "Distributed deployment algorithms for efficient coverage in a network of mobile sensors with nonidentical sensing capabilities," *IEEE Transactions on Vehicular Technology*, vol. 63, no. 8, pp. 3998–4016, Oct 2014.

[9] F. Sharifi, A. Chamseddine, H. Mahboubi, Y. Zhang, and A. Aghdam, "A distributed deployment strategy for a network of cooperative autonomous vehicles," *Control Systems Technology, IEEE Transactions on*, vol. 23, no. 2, pp. 737–745, March 2015.

[10] M. Pavone, E. Frazzoli, and F. Bullo, "Adaptive and distributed algorithms for vehicle routing in a stochastic and dynamic environment," *Automatic Control, IEEE Transactions on*, vol. 56, no. 6, pp. 1259–1274, June 2011.

[11] A. Okabe, B. Boots, and K. Sugihara, *Spatial Tessellations: Concepts and Applications of Voronoi Diagrams*. New York, NY, USA: John Wiley & Sons, Inc., 1992.

[12] M. A. Guvensan and A. G. Yavuz, "On coverage issues in directional sensor networks: A survey," *Ad Hoc Networks*, vol. 9, no. 7, pp. 1238 – 1255, 2011.

[13] O. Tekdas and V. Isler, "Sensor placement algorithms for triangulation based localization," in *Robotics and Automation, 2007 IEEE International Conference on*, April 2007, pp. 4448–4453.

[14] D. Tao and T. Y. Wu, "A survey on barrier coverage problem in directional sensor networks," *IEEE Sensors Journal*, vol. 15, no. 2, pp. 876–885, Feb 2015.

[15] L. C. A. Pimenta, V. Kumar, R. C. Mesquita, and G. A. S. Pereira, "Sensing and coverage for a network of heterogeneous robots," in *Decision and Control, 2008. CDC 2008. 47th IEEE Conference on*, Dec 2008, pp. 3947–3952.

[16] H. G. Tanner and D. K. Christodoulakis, "Cooperation between aerial and ground vehicle groups for reconnaissance missions," in *Decision and Control, 2006 45th IEEE Conference on*, Dec 2006, pp. 5918–5923.

[17] B. Grocholsky, J. Keller, V. Kumar, and G. Pappas, "Cooperative air and ground surveillance," *IEEE Robotics Automation Magazine*, vol. 13, no. 3, pp. 16–25, Sept 2006.

[18] F. Abbasi, A. Mesbahi, and J. Mohammadpour, "Team-based coverage control of moving sensor networks," in *2016 American Control Conference (ACC)*, July 2016, pp. 5691–5696.

[19] M. Schwager, D. Rus, and J.-J. Slotine, "Decentralized, adaptive coverage control for networked robots," *Int. J. Rob. Res.*, vol. 28, no. 3, pp. 357–375, 2009.