Distributed observer-based cooperative control for output regulation in multi-agent linear parameter-varying systems

Afshin Mesbahi1, Javad Mohammadpour Velni1

1School of Electrical & Computer Engineering, College of Engineering, The University of Georgia, Athens, GA 30602, USA
2E-mail: javadm@uga.edu

Abstract: In this paper, output regulation problem is examined for a class of heterogeneous multi-agent systems, whose dynamics are governed by affine linear parameter-varying (LPV) models, with a known communication topology. In the proposed solution method, the agents are divided into two groups depending on whether or not their output is directly affected by external (such as reference or disturbance) inputs. Conditions for cooperative LPV control synthesis are first constructed through the design of distributed observers. To establish these conditions, the solution to a time-varying Sylvester equation is required. An offline solution to this equation will be proposed to calculate and update the controller state-space matrices. The proposed design methodologies of this study are finally validated using a numerical example.

1 Introduction

The output regulation problem, that is one of the fundamental problems in control systems theory, aims to achieve asymptotic tracking of reference trajectories and/or asymptotic rejection of disturbances generated by an external autonomous system (called exosystem) [1, 2]. Multi-agent systems have attracted significant attention over the past few years due to their various applications [3–12]. The output regulation problem has also been thoroughly investigated in the literature for linear multi-agent systems [13–16]. The output regulation problem can be viewed as a generalisation of the leader-following consensus, formation and rendezvous problems [17–20]. A decentralised full information control scheme has been proposed in [14, 21] for the output synchronisation problem in linear networked systems, where all the contributing nodes in the underlying graph are assumed to be identical. The results have been extended for non-identical linear multi-agent systems in [22]. A distributed observer-based control law has been proposed in [23] to address the output regulation problem for linear heterogeneous multi-agent systems.

The presence of non-linearities in the system dynamics is another challenging issue for the control design problem in multi-agent systems. Furthermore, in many practical multi-agents applications, the agents often exhibit time varying dynamics due to the variation in endogenous or exogenous parameters [24–28]. Stability analysis and control design problems for linear parameter-varying (LPV) systems have been studied to cope with the non-linearities of such physical systems. Unlike significant developments in cooperative output regulation for linear multi-agent systems, there has been a very limited amount of work dedicated to multi-agent LPV systems. The state consensus problem has been addressed in [29] for a class of (both homogeneous and heterogeneous) multi-agent LPV systems with affine dependency on LPV parameters, in which the full knowledge of states is required for the controller design. The output synchronisation problem has also been studied in multi-agent LPV systems with heterogeneous parameters [30], in which the equivalent graph representation of the agents formation is assumed to be uniformly connected. The aforementioned articles consider multi-agent systems without any exogenous disturbance inputs. The synchronisation problem has been recently addressed in [31] for the heterogeneous affine LPV systems, in which all agents are assumed to have access to the exogenous input. A sufficient condition has also been given for solving the gain-scheduled leader-follower tracking control problem [32]. The proposed control law requires the full knowledge of states of each agent and of the leader.

Motivated by the recent afore-described advancements and the existing gaps in the cooperative output regulation of multi-agent LPV systems, this paper provides design methods for distributed cooperative output regulation of heterogeneous multi-agent LPV systems. The communication topology between agents is assumed to be described by a directed graph. Realistically, some of the agents may only communicate with others and do not necessarily have direct access to the external input signal. In addition, there exists a directed spanning tree rooted at the external input signals. The agents are classified into informed and uninformed groups, where agents of the informed group can locally reconstruct the exosystem signals. However, reconstructing the exosystem signals for agents of the uninformed group is dependent on information they exchange with the informed group. From a practical point of view, states of the exosystem or even of the agents are usually unavailable for the implementation of control laws. Therefore, a distributed observer design is proposed to estimate the states of agents and the exosystem. The parameter-varying controllers are designed under the assumption that each agent is decoupled from others, while each agent's controller is able to access only its own outputs and its neighbours. A condition is first established to ensure the solvability of the output regulation problem in multi-agent LPV systems. The obtained condition is in the form of a so-called Sylvester equation with time-varying coefficients. Gradient-based recurrent neural networks were previously used to solve time-varying Sylvester equations; but they led to an error in finding the solution [33, 34]. This estimation error can be reduced to zero by applying recurrent neural networks [33] that have restrictions for real time implementation since the estimation error does not converge to zero in finite time [35]. The estimation error can, however, converge to zero in a finite time if sign-bi-power or Li activation functions are used [35, 36], but the estimated upper bound of the convergence time is conservative [37]. In this paper, an analytical solution to a time-varying Sylvester equation is obtained for the special case that time-varying coefficients of the Sylvester equation have affine structures. Solution to the time-varying Sylvester equation is computed simply by solving a set of linear algebraic equations. A condition is also obtained to guarantee the solvability of the output regulation problem when the models of multi-agent LPV system have an affine structure.

The remaining of this paper is structured as follows. Multi-agent systems described by LPV models are presented in Section 2. The main results of this paper are presented in Section 3. Sufficient
conditions are provided to guarantee that the cooperative output regulation problem is solvable for multi-agent LPV systems. The obtained conditions are then simplified for multi-agent LPV systems with an affine structure. In Section 4, a numerical example is given to demonstrate the effectiveness of the proposed control design method. Concluding remarks are finally given in Section 5.

Notation: Throughout this paper, we assume that $\mathbf{R}$, $\mathbf{I}$, $\mathbf{A}^T$, $\text{diag}(\mathbf{A}, \mathbf{B})$, $\text{col}(\mathbf{A}, \mathbf{B})$, $\text{vec}(\mathbf{A})$, $\mathbf{A} \otimes \mathbf{B}$ and $\mathbf{A} \oplus \mathbf{B}$ denote the set of real numbers, the identity matrix of appropriate dimension, the transpose of $\mathbf{A}$, the block diagonal matrix with block diagonals $\mathbf{A}$ and $\mathbf{B}$, $[\mathbf{A}^T, \mathbf{B}^T]^T$, the vectorisation of $\mathbf{A}$, the Kronecker product of $\mathbf{A}$ and $\mathbf{B}$ and the Kronecker sum of $\mathbf{A} \in \mathbb{R}^{n \times n}$ and $\mathbf{B} \in \mathbb{R}^{m \times m}$ defined as $\mathbf{A} \otimes \mathbf{B} = \mathbf{A} \otimes \mathbf{I}_m + \mathbf{I}_n \otimes \mathbf{B}$.

2 Preliminaries and problem statement

In this section, we first provide a brief description of the fundamentals of graph theory. We then describe the dynamic models governing the multi-agent systems and finally give a description of the problem addressed in this paper.

2.1 Fundamentals of graph theory

In the context of multi-agent systems, we use a (directed or undirected) graph denoted by $G = (\mathcal{V}, \mathcal{E})$ to model the communication among the agents, where $\mathcal{V} = \{1, \ldots, N\}$ denotes the set of nodes and $\mathcal{E} \subseteq \mathcal{V} \times \mathcal{V}$ denotes the edge set. In an undirected graph, $(i, j) \in \mathcal{E}$ implies that $(j, i) \in \mathcal{E}$. Agent $i$ is said to have access to the information of agent $j$ when $(i, j) \in \mathcal{E}$, and agent $j$ is also called the neighbour of agent $i$. The neighbourhood of the node $i$ is defined as $\mathcal{N}_i = \{j \in \mathcal{V} | (j, i) \in \mathcal{E}\}$. A directed path from node $i$ to node $j$ is a sequence of ordered edges of the form $(i, n_1, n_2, \ldots, n_r, j)$. A graph has a directed spanning tree rooted at node $i$ if there is a directed path from the node $i$ to all other nodes.

Non-negative matrix $\mathcal{M} = [a_{ij}] \in \mathbb{R}^{N \times N}$ is a weighted adjacency matrix of digraph $G$ if $a_{ii} = 0$ and $a_{ij} > 0$ for $(i, j) \in \mathcal{E}$. Finally, Laplacian matrix $L = [l_{ij}] \in \mathbb{R}^{N \times N}$ of digraph $G$ is defined as $l_{ii} = \sum_{j \neq i} a_{ii} l_{ij}$ and $l_{ij} = -a_{ij} > 0$ for $i \neq j$.

In the remaining of this paper, the exosystem, i.e. dynamic system that generates the external input signals, is considered as node 0. Define graph $G = (\mathcal{V}, \mathcal{E})$, where $\mathcal{V} = \{0, 1, \ldots, N\}$. Let $\Delta = \text{diag}(a_{01}, \ldots, a_{0N})$ where $a_{0i} > 0$ for $i \in \{1, \ldots, N\}$ and $a_{00} = 0$ for $k \in \{M + 1, \ldots, N\}$. Define Laplacian matrix $\mathcal{L}$ as follows, where matrix $-\mathcal{L}$ is a Metzler matrix with zero row sum.

$$
\mathcal{L} = \begin{bmatrix}
0 & \mathcal{M} \\
-\mathcal{M} & -\mathcal{L} + \Delta
\end{bmatrix}
$$

2.2 State-space representation of the agents

We consider $N$ multi-agent dynamic systems, each represented by an LPV model as

$$
\begin{align*}
\dot{x}_k(t) &= A_k(\rho_k(t))x_k(t) + B_ku_k(t) + E_k(\rho_k(t))d(t), \\
y_k(t) &= C_k(\rho_k(t))x_k(t) + D_ku_k(t) + F_k(\rho_k(t))d(t), \\
e_k(t) &= G_k(\rho_k(t))x_k(t) + R_ku_k(t) + T_k(\rho_k(t))d(t),
\end{align*}
$$

(1)

for $k \in \mathcal{J} = \{1, \ldots, N\}$, where $x_k(t) \in \mathbb{R}^{n_k}$ is the state vector, $y_k(t) \in \mathbb{R}^{n_y}$ is the measurement outputs vector, $e_k(t) \in \mathbb{R}^{n_e}$ is the vector of regulated outputs, e.g. tracking error, and $u_k(t) \in \mathbb{R}^{n_u}$ is the vector of control inputs for agent $k$. Also, $d(t) \in \mathbb{R}^{n_d}$ denotes the exogenous signal. Unlike the system matrices $B_k$, $D_k$ and $R_k$ that are assumed to be constant, the matrices $A_k$, $C_k$, $G_k$, $E_k$, $F_k$ and $T_k$ could in general depend on the time-varying parameters $\rho_k(t)$ (referred to as ‘scheduling variables’), which are bounded and measurable in real time. Matrices $E_k$, $F_k$ and $T_k$ are given based on the agents, the exosystem, and defined regulated outputs. We further assume that $\rho_k(t) \in \mathcal{P}_k$, where $\mathcal{P}_k$ denotes a $p_k$-dimensional admissible set of scheduling variables. Even in the case that the agents share the same dynamics structure, i.e. number of states, inputs and outputs, the agents can be heterogeneous due to the difference in the local scheduling variables $\rho_k(t)$. An linear time-invariant (LTI) counterpart of (1) has been studied in [20, 22, 23, 38–41].

The exogenous signal $d(t)$ may represent the disturbance to be rejected or the reference input to be tracked, and is described by the following model (which is a standard representation in the literature, see e.g. [17–20, 21, 23, 42–44])

$$
d(t) = Sd(t).
$$

(2)

Describing the exosystem by (2) makes a distinction between the output regulation problem and the standard trajectory tracking problem. While the trajectory is considered to be known in the trajectory tracking problem, in the output regulation problem, a disturbance signal to be rejected or a reference input to be tracked is generated by a given exosystem described by (2). The exosystem (2) can be considered as a leader in a leader-follower multi-agent system architecture. The agents described by (1) are classified into informed agents and uninformed agents. The informed agents are those that are informed by the leader, i.e. have access to the exosystem information, while the exogenous signal is not accessible by uninformed agents. Assume, without the loss of generality, that the agent $i$ is an informed agent for $i \in \mathcal{J} = \{1, \ldots, M\}$, and the agent $j$ is an uninformed agent for $j \in \mathcal{J} = \{M + 1, \ldots, N\}$. According to the definition, the coefficient matrix of the exogenous signal $d(t)$ in (1) is given by

$$
\begin{bmatrix}
F_k(\rho_k(t)) & \neq 0, & \text{if } k \in \mathcal{J} \\
F_k(\rho_k(t)) & = 0, & \text{if } k \in \mathcal{J}.
\end{bmatrix}
$$

(3)

Remark 1: As discussed before, the system matrices $B_k$, $D_k$ and $R_k$ in (1) are assumed to be parameter-independent. Generally, a system with parameter-varying $B_k$, $D_k$ or $R_k$ can be transformed into the form (1) by applying a low-pass filter to the control input [45].

2.3 Cooperative output regulation problem for multi-agent LPV systems

For each agent, we consider a parameter-varying controller with the following structure:

$$
x_{\chi_k}(t) = A_{\chi_k}(\rho_k(t))x_k(t) + B_{\chi_k}(\rho_k(t))y_k(t) + \sum_{i \in \mathcal{J}_k} A_{\chi_i}(\rho_i(t))x_{\chi_i}(t) + C_{\chi_i}(\rho_i(t))x_{\chi_i}(t),
$$

(4)

where $x_{\chi_k}(t)$ is the estimate of the augmented vector of open-loop system states and the exogenous signal $(x_{\chi_k}^T \triangleq \text{col}(x_{\chi_k}(t), d(t)))$, i.e. $x_{\chi_k} = \mathcal{L}x_{\chi_k}$. The LPV controllers above use the local state estimates $(x_{\chi_k} = \tilde{x}_{\chi_k})$ and the controller matrices $A_{\chi_i}$, $B_{\chi_i}$, $A_{\chi_i}$ and $C_{\chi_i}$ are to be determined. We aim at designing the controllers of the above structure to ensure that the following two objectives are satisfied.

Objective 1 (internal stability): Assume $d(t) = \mathbf{0}$ and $u(t) = \mathbf{0}$ for $k \in \mathcal{J}$. For all initial conditions $x_k(0) = x_{k,0}$, $x_k(0) = x_{k,0}$ and $\rho_k \in \mathcal{P}_k$, we should have...
describe a few standard assumptions imposed on the agents
dynamics. It is noted that the following assumptions are standard
one that have also been considered in the previous relevant studies
[1, 23, 29, 30, 46, 47].

**Assumption 1**: The pair \( (A_k(\rho_k(t)), B_k) \) is stabilisable for any \( k \in \mathcal{N} \) and \( \rho_k \in \mathcal{R}_k \).

**Assumption 2**: The exosystem (2) is not asymptotically stable.

It is noted that Assumption 2 can be relaxed without the loss of
generality. In fact, Objective 1 is only dependent on the system
matrices and independent of the exosystem. In addition, if the
closed-loop system with a designed controller satisfies Objectives
1 and 2 under Assumption 2, then the two objectives are also
satisfied by the same controller even if Assumption 2 is relaxed.

**Assumption 3**: The pair

\[
\begin{bmatrix}
A_k(\rho_k(t)) & E_k(\rho_k(t)) \\
0 & S_k
\end{bmatrix}, \quad \begin{bmatrix}
C_k(\rho_k(t)) & F_k(\rho_k(t))
\end{bmatrix}
\]

is detectable for any \( k \in \mathcal{J} \) and \( \rho_k \in \mathcal{R}_k \).

**Assumption 4**: The pair \( (C_k(\rho_k(t)), A_k(\rho_k(t))) \) is detectable for any \( k \in \mathcal{U} \) and \( \rho_k \in \mathcal{R}_k \).

**Assumption 5**: Communication digraph \( \mathcal{G} \) contains a directed
spanning tree rooted at 0. Assumption 1 together with Assumptions 3 and 4 make it
possible to guarantee that the multi-agent LPV system is detectable and
stable. It is noted that these assumptions are essential since otherwise Objectives 1 and 2 would not be satisfied by an
output feedback controller. Subsystems (5) and (6) below represent the ‘informed group’ and are constructed by considering the
models of informed agents as

\[
x^f(t) = A^f(\rho^f(t))x^f(t) + B^f u^f(t) + E^f(\rho^f(t))d^f(t),
\]

\[
y^f(t) = C^f(\rho^f(t))x^f(t) + D^f u^f(t) + F^f(\rho^f(t))d^f(t),
\]

\[
e^f(t) = G^f(\rho^f(t))x^f(t) + R^f u^f(t) + T^f(\rho^f(t))d^f(t),
\]

and

\[
\lim_{t \to \infty} x_k(t) = 0 \quad \text{and} \quad \lim_{t \to \infty} x_{i_k}(t) = 0, \quad k \in \mathcal{N}.
\]
The closed-loop interconnection of the above controller with the
where (see equation below) For the LTI case, Lemma 1.4 in [1]
Assumption 2 and Objective 1 are satisfied, there exists a unique
Fig. 1
configuration of the closed-loop system
\[ \begin{align*}
\dot{x}^*(t) &= \begin{bmatrix}
\xi^*(t) \\
\zeta^*(t)
\end{bmatrix} = A^*_\rho^*(r^*(t))\xi^*(t) + B^*_\rho^*(r^*(t))y^*(t), \\
\rho^*(t) &= C^*_\rho^*(r^*(t))\xi^*(t) + D^*_\rho^*(r^*(t))y^*(t).
\end{align*} \]
(14)

The closed-loop interconnection of the above controller with the
augmented system (5) leads to (see (15))
\[ e^*(t) = \begin{bmatrix}
G^*(r^*(t)) & R^*C^*_\rho^*(r^*(t))
\end{bmatrix}
\begin{bmatrix}
\dot{x}^*(t) \\
\dot{\xi}^*(t)
\end{bmatrix}
+ T^*(r^*(t))d^*(t) = C^*_\rho^*(r^*(t))x^*_d(t)
+ D^*_\rho^*(r^*(t))d^*(t),
\]
(16)
where (see equation below) For the LTI case, Lemma 1.4 in [1]
relates Objectives 1 and 2. This lemma is extended to the LPV
systems in the following lemma.

**Lemma 1:** Consider Assumption 2 and suppose that the closed-
loop system represented by (15) and (16) satisfies Objective 1. Then,
Objective 2 is satisfied as well if there exists a matrix \( \Theta \)
satisfying the following set of matrix equations for any admissible \( r(t) \),
\begin{align*}
A^*_\rho^*(r^*(t))\Theta - \Theta S^* &= B^*_\rho^*(r^*(t)) = 0, \\
C^*_\rho^*(r^*(t))\Theta + D^*_\rho^*(r^*(t)) &= 0.
\end{align*}
(17a, 17b)

**Proof:** Equation (17a) is the so-called Sylvester equation. Since
Assumption 2 and Objective 1 are satisfied, there exists a unique
matrix \( \Theta \) which renders the Sylvester equation (17a) to have a
solution (see [34], which has proven this for time-varying Sylvester
equation). The following equations are obtained by substituting
\( \xi(t) \oplus x^*_d(t) - \Theta d^*(t) \) into the closed-loop system representation
(15) and (16) and considering (17a)
\begin{align*}
\xi(t) &= A^*_\rho^*(r^*(t))\xi(t) + \left( A^*_\rho^*(r^*(t))\Theta - \Theta S^* \right) \\
&+ B^*_\rho^*(r^*(t))d^*(t) = A^*_\rho^*(r^*(t))\xi(t),
\end{align*}
(18)
\begin{align*}
e^*(t) &= C^*_\rho^*(r^*(t))\xi(t) \\
&+ (C^*_\rho^*(r^*(t))\Theta + D^*_\rho^*(r^*(t)))d^*(t).
\end{align*}

According to Objective 1, \( \xi(t) \) is asymptotically stable, and hence
\( \lim_{t \to \infty} \xi(t) = 0 \). Since the matrix \( \Theta \) satisfies (17b), then using
(18), \( \lim_{t \to \infty} e^*(t) = 0 \). □

Lemma 1 shows that Objectives 1 and 2 are related. Next
lemma establishes this relation through employing the system
matrices in (5), ecossystem (6) and controller (14), and extends
Lemma 1.13 in [1] to the LPV case.

**Lemma 2:** Suppose that Assumption 2 is satisfied and that the
closed-loop system (15) and (16) satisfies Objective 1. Then,
Objective 2 is satisfied as well if there exist matrices \( \Pi^* \),
\( \Gamma^*(r^*(t)) \), and \( \Upsilon \) such that
\begin{align*}
A^*(r^*(t))\Pi^* + B^*(r^*(t))\Gamma^*(r^*(t)) &+ E^*(r^*(t)) = \Pi^*S^*, \\
A^*_\rho^*(r^*(t))\Upsilon + B^*_\rho^*(r^*(t))C^*(r^*(t))\Pi^* \\
&+ D^*_\rho^*(r^*(t)) + F^*(r^*(t)) = \Upsilon S^*.
\end{align*}
(19a, 19b)
\begin{align*}
G^*(r^*(t))\Pi^* + R^*(r^*(t)) + T^*(r^*(t)) &= 0,
\end{align*}
(19c)

The closed-loop interconnection of the above controller with the
augmented system (5) leads to (see (15))
\[ \begin{align*}
\begin{bmatrix}
\dot{x}^*_d(t) \\
\dot{\xi}^*_d(t)
\end{bmatrix} &= \begin{bmatrix}
A^*_\rho^*(r^*(t)) & B^*_\rho^*(r^*(t))C^*_\rho^*(r^*(t)) \\
B^*_\rho^*(r^*(t))C^*_\rho^*(r^*(t)) & A^*_\rho^*(r^*(t)) + B^*_\rho^*(r^*(t))D^*_\rho^*(r^*(t))
\end{bmatrix}
\begin{bmatrix}
x^*_d(t) \\
\xi^*_d(t)
\end{bmatrix}
+ \begin{bmatrix}
E^*(r^*(t)) \\
F^*(r^*(t))
\end{bmatrix}d^*(t),
\end{align*} \]
(15)

\begin{align*}
x^*_d(t) &= \begin{bmatrix}
\dot{x}^*_d(t) \\
\dot{\xi}^*_d(t)
\end{bmatrix},
\end{align*}
\begin{align*}
A^*_\rho^*(r^*(t)) &= \begin{bmatrix}
A^*(r^*(t)) & B^*(r^*(t))C^*(r^*(t))
\end{bmatrix},
B^*_\rho^*(r^*(t))C^*_\rho^*(r^*(t)) &= \begin{bmatrix}
A^*_\rho^*(r^*(t)) & B^*_\rho^*(r^*(t))D^*_\rho^*(r^*(t))
\end{bmatrix},
\end{align*}
\begin{align*}
C^*_\rho^*(r^*(t)) &= \begin{bmatrix}
G^*(r^*(t)) & R^*C^*(r^*(t))
\end{bmatrix},
D^*_\rho^*(r^*(t)) &= \begin{bmatrix}
T^*(r^*(t))
\end{bmatrix}.
\end{align*}
where \( \Gamma^F(\rho^F(t)) = C^T_F(\rho^F(t))Y \).

**Proof:** Substituting \( \Gamma^F(\rho^F(t)) = C^T_F(\rho^F(t))Y \) into subequations (19) results in

\[
A^F(\rho^F(t))\Pi^F + B^F C^F(\rho^F(t))Y + E^F(\rho^F(t)) = \Pi^F S^F , \tag{20a}
\]

\[
B^F(\rho^F(t))C^F(\rho^F(t))\Pi^F + B^F(\rho^F(t))F^F(\rho^F(t)) + (A^F(\rho^F(t)) + B^F(\rho^F(t))D^F C^F(\rho^F(t)))Y = YS^F , \tag{20b}
\]

\[
G^F(\rho^F(t))\Pi^F + R^F C^F(\rho^F(t))Y + T^F(\rho^F(t)) = 0 . \tag{20c}
\]

which give (21a) and (19a)–(19c) and by solving the matrix inequality problems (7) and (8).

Next, we define the matrix decomposition \( \Theta \triangleq \text{col}(\Pi^F, Y) \). Equations (17a) and (17b) are, respectively, obtained by substituting \( \Theta \) in (21a) and (21b). Then, according to Lemma 1, Objective 1 is satisfied, and this completes the proof. \( \square \)

In the remainder of this section, we describe the process of obtaining the controller state-space matrices to guarantee that the resulting closed-loop system meets Objectives 1 and 2. To this end, we consider that the state-space matrices of the controller (14) are constructed as follows:

\[
A^d(\rho^d(t)) = \begin{bmatrix} A^F(\rho^F(t)) & E^F(\rho^F(t)) & B^F \\ 0 & S^F & 0 \\ K^F & K^F(\rho^F(t)) + L^F \end{bmatrix}, \tag{22}
\]

\[
B^d(\rho^d(t)) = -L^F, \quad C^d(\rho^d(t)) = \begin{bmatrix} K^F & K^F(\rho^F(t)) \end{bmatrix}, \quad D^d(\rho^d(t)) = 0 .
\]

where \( K^F(\rho^F(t)) \triangleq \Gamma^F(\rho^F(t)) - K^F \Pi^F \) and matrices \( \Pi^F \), \( \Gamma^F(\rho^F(t)) \), \( K^F \) and \( \text{col}(L^F, L^F) \) are, respectively, obtained from (19a)–(19c) and by solving the matrix inequality problems (7) and (8).

In the case of LTI systems, a necessary and sufficient condition has been established in Theorem 2.4.1 in [47] and Theorem 1.14 in [1] to guarantee Objectives 1 and 2. We extend those results to the LPV systems in the following lemma.

**Lemma 3:** Suppose that Assumptions 1–3 are satisfied. The closed-loop system (15) and (16) associated with the augmented multi-agent system (5) and the augmented controller (14) with the state-space matrices given in (22) satisfies Objectives 1 and 2 if there exist matrices \( \Pi^F \) and \( \Gamma^F(\rho^F(t)) \) as solutions to the following algebraic equations:

\[
A^F(\rho^F(t))\Pi^F + B^F T^F(\rho^F(t)) + E^F(\rho^F(t)) = \Pi^F S^F , \tag{23a}
\]

\[
G^F(\rho^F(t))\Pi^F + R^F T^F(\rho^F(t)) + T^F(\rho^F(t)) = 0 . \tag{23b}
\]

**Proof:** Suppose that the matrices \( \Pi^F \) and \( \Gamma^F(\rho^F(t)) \) are solutions to (23). Defining \( \hat{A}_d(\rho^d(t)) \) as

\[
\hat{A}_d(\rho^d(t)) = \begin{bmatrix} 1 & 0 & 0 \\ -1 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}, \quad A^d(\rho^d(t)) = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}
\]

results in the following through some algebraic manipulations (see equation below) with \( A^d \) defined in (9). Due to the structure of \( A^d \) (that implies that it is asymptotically stable), there exists a symmetric positive definite matrix \( \hat{Q} \) such that

\[
\hat{Q} \hat{A}_d^T(\rho^d(t)) + \hat{A}_d(\rho^d(t)) \hat{Q} < 0 .
\]

On the other hand, there exists a symmetric positive definite matrix \( \hat{Q} \) such that \( \hat{Q} \hat{A}_d^T(\rho^d(t)) + \hat{A}_d(\rho^d(t)) \hat{Q} < 0 \). This implies that the controller with the state-space matrices given by (22) satisfies Objective 1. Defining \( Y = \text{col}(\Pi^F, I) \) and post-multiplying \( A^d \) by \( Y \) results in (see equation below) Employing (22) and (23a) results in

\[
A^d(\rho^d(t))Y = \begin{bmatrix} \Pi^F \\ Y \end{bmatrix} S^F - B^d(\rho^d(t)) \begin{bmatrix} C^F(\rho^F(t))\Pi^F + F^d(\rho^F(t)) + D^F \Gamma^F(\rho^F(t)) \\ \end{bmatrix}
\]

which gives (19b). Hence, according to Lemma 2, \( \lim_{t \to \infty} \varepsilon^F(t) = 0 \) and this completes the proof. \( \square \)

Next, let us consider that the following controller is applied to the uninformed group

\[
\hat{A}_d(\rho^d(t)) = \begin{bmatrix} A^F(\rho^F(t)) + B^F K^F_1 \\ 0 \\ 0 \end{bmatrix}, \quad \hat{B}_d(\rho^d(t)) = \begin{bmatrix} B^F K^F_1 \\ B^F K^F_2 \end{bmatrix}, \quad \hat{C}_d(\rho^d(t)) = \begin{bmatrix} A^F(\rho^F(t)) + B^F K^F_1 \\ L^F \end{bmatrix}, \quad \hat{D}_d(\rho^d(t)) = \begin{bmatrix} B^F K^F_1 \\ B^F K^F_2 \end{bmatrix}
\]

\[
\begin{bmatrix} A^F(\rho^F(t)) + B^F K^F_1 \\ 0 \end{bmatrix}
\]
\[ \xi(t) = \begin{bmatrix} \xi^W(t) \\ \xi^U(t) \end{bmatrix} = A^W(\rho^W(t))\xi^W(t) + A^U(\rho^U(t))\xi^U(t) + B^W(\rho^W(t))y(t), \]

where the state-space matrices of the controller (24) are constructed as

\[ A^W(\rho^W(t)) = \begin{bmatrix} A^W(\rho^W(t)) & E^W(\rho^W(t)) \\ 0 & \{I \otimes S^W\} - \mu(\mathcal{Z}_3 \otimes I) \end{bmatrix}, \]

\[ B^W(\rho^W(t)) = \begin{bmatrix} L^W \end{bmatrix}, \]

\[ C^W(\rho^W(t)) = \begin{bmatrix} K^W \\ K^U(\rho^U(t)) \end{bmatrix}, \]

\[ A^U(\rho^U(t)) = \begin{bmatrix} 0 \\ 0 \end{bmatrix}, \]

\[ B^U(\rho^U(t)) = -\begin{bmatrix} L^U \end{bmatrix}, \]

\[ C^U(\rho^U(t)) = \begin{bmatrix} K^U_d \\ K^U(\rho^U(t)) \end{bmatrix}, \]

in which \( \mu \) is a positive real scalar.

**Lemma 4:** Suppose that Assumptions 1, 2, 4 and 5 are satisfied and \( \lim_{t \to \infty} \xi^U(t) = d^U(t) \). The closed-loop system associated with the augmented multi-agent system (10) and the augmented controller (24) with the state-space matrices given in (25) satisfies Objectives 1 and 2 if there exist matrices \( \Pi^W \) and \( \Pi^U(\rho^U(t)) \) that are solutions to the following algebraic equations:

\[ A^W(\rho^W(t))\Pi^W + B^W(\rho^W(t)) + E^W(\rho^W(t)) = \Pi^W S^W, \]

\[ G^W(\rho^W(t))\Pi^W + R^W(\rho^W(t)) + T^W(\rho^W(t)) = 0. \]

**Proof:** Using (24) and (25), it is easy to obtain the following:

\[ \xi^W(t) = \begin{bmatrix} \xi^W(t) \\ \xi^U(t) \end{bmatrix} = A^W(\rho^W(t))\xi^W(t) + A^U(\rho^U(t))\xi^U(t) + B^W(\rho^W(t))y(t) + \Pi^U(\rho^U(t))\xi^U(t) \]

and

\[ \xi^W(t) = \begin{bmatrix} \xi^W(t) \\ \xi^U(t) \end{bmatrix} = \begin{bmatrix} A^W(\rho^W(t)) & E^W(\rho^W(t)) + B^W(\rho^W(t))K^W \end{bmatrix} \xi^W(t) + L^W \xi^U(t). \]

Define \( S^W = (-\mu(\mathcal{Z}_3) \otimes S^W) \). Equation (28) can then be rewritten as

\[ \xi^W(t) = S^W \xi^W(t) - \mu(\mathcal{Z}_3 \otimes I)\xi^U(t). \]

Let \( \lambda_j(S^W) \) and \( \lambda_j(\mathcal{Z}_3) \) are, respectively, the \( j \)th eigenvalue of \( S^W \) and the \( j \)th eigenvalue of \( \mathcal{Z}_3 \). Then \( \lambda_j(S^W) - \mu_j(\mathcal{Z}_3) \) is an eigenvalue of \( S^W \) (see p. 412 in [48]). According to Assumption 5, the real part of the eigenvalues of \( \mathcal{Z}_3 \) is positive based on Lemma 1 in [23]. Therefore, \( S^W \) is a Hurwitz matrix for sufficiently large \( \mu \). Through some algebraic manipulations, (29) can be rewritten as

\[ \xi^W(t) = S^W \xi^W(t) - d^W(t), \]

\[ -\mu(\mathcal{Z}_3 \otimes I)\xi^U(t) = d^U(t). \]

Since \( \lim_{t \to \infty} \xi^U(t) = d^U(t) \) and \( S^W \) is a Hurwitz matrix, it is concluded that \( \lim_{t \to \infty} \xi^U(t) = d^U(t) \). Also, since \( \lim_{t \to \infty} \xi^W(t) = d^W(t) \), the system represented by (27) is a special case of the system (14). The rest of the proof is similar to proof of Lemma 1.

Augmenting controllers (14) and (24) with the matrices given in (22) and (25) results in a distributed controller with the following structure:

\[ \begin{bmatrix} \xi^W(t) \\ \xi^U(t) \end{bmatrix} = A^W(\rho(t))\xi^W(t) + B^W(\rho(t))y(t), \]

\[ u(t) = C^W(\rho(t))\xi^W(t). \]

\[ \begin{bmatrix} A^W(\rho(t)) & 0 \\ B^W(\rho(t)) & D^W(\rho(t)) \end{bmatrix} = \begin{bmatrix} A^U(\rho(t)) & 0 \\ B^U(\rho(t)) & D^U(\rho(t)) \end{bmatrix}, \]

\[ \begin{bmatrix} C^W(\rho(t)) & 0 \\ 0 & C^U(\rho(t)) \end{bmatrix} \]

where \( \rho(t) = \text{col}(\rho^W, \rho^U) \).

**Theorem 1:** Suppose that Assumptions 1–5 are all satisfied. Then, the controller represented by (30) satisfies Objectives 1 and 2 if the algebraic equations (26) and (23) have a solution.

**Proof:** Suppose that (23) has a solution. Since the conditions of Lemma 3 are met, \( \lim_{t \to \infty} \xi^W(t) = 0 \) and \( \lim_{t \to \infty} \xi^U(t) = 0 \). Considering that \( \xi(t) = x(t) - \Theta \dot{x}(t) \) in the proof of Lemma 1, it is concluded that \( \lim_{t \to \infty} \xi^W(t) = d^W(t) \). Therefore, \( \lim_{t \to \infty} \xi^U(t) = 0 \) because conditions of Lemma 4 are satisfied.

3.1 Solvability condition for the regulation problem

To satisfy Objectives 1 and 2, (23) and (26) in Lemmas 3 and 4 are needed to be solved. These are time-varying equations, which provide an infinite number of linear equations. To convert the problem to a finite number of linear equations, we restrict our study to affine LPV systems. Assume that time-varying matrices of system (1) have an affine dependency on the scheduling variables as

\[ \Omega(t) = \Omega^W + \sum_{k=1}^{\rho} \rho^j_k(t) \Omega^j_k, \]

where \( \rho^j_k(t) \) are non-zero and non-negative continuous functions, for \( k \in \mathcal{N} \) and \( \Omega \in \{A, C, E, F, G, R, T\} \). In addition, \( \rho^j_k(t) \) is a Hurwitz matrix.
represents the $i$th element of the vector of scheduling variables $\rho_i(t)$.

Corollary 1: Suppose that Assumptions 1–3 are satisfied. Closed-loop system (15) and (16) associated with the augmented multi-agent systems (5) and (10) with affine structure (31) and augmented controller (30) satisfies Objectives 1 and 2 if there exist matrices $\Pi$, $\Gamma$, ..., $\Gamma_k$ that satisfy (32) for $k \in \mathcal{N}$.

\[
\begin{bmatrix}
(-S) \otimes A_i & I_{n_x} \otimes B_k & 0 & \ldots \\
I_{n_x} \otimes G_i & I_{n_x} \otimes R_k & 0 & \ldots \\
I_{n_x} \otimes A_i & I_{n_x} \otimes B_k & 0 & \ldots \\
I_{n_x} \otimes G_i & I_{n_x} \otimes R_k & 0 & \ldots \\
\vdots & \vdots & \vdots & \ddots \\
I_{n_x} \otimes A_i & I_{n_x} \otimes B_k & 0 & \ldots \\
I_{n_x} \otimes G_i & I_{n_x} \otimes R_k & 0 & \ldots \\
\end{bmatrix} 
\begin{bmatrix}
I_{n_x} \\
I_{n_x} \\
I_{n_x} \\
I_{n_x} \\
\vdots \\
I_{n_x} \\
I_{n_x} \\
\end{bmatrix} = 0
\]

(32)

**Proof:** Assume that $\Pi$ and $\Gamma(\rho(t))$ are constructed by $\Pi = \text{diag}(\Pi_1, ..., \Pi_k)$ and $\Gamma(\rho(t)) = \text{diag}(\Gamma_1(\rho(t)), ..., \Gamma_k(\rho(t)))$.

Due to the structure of the state-space matrices of the augmented system (5) and (26) are equivalent with

\[
A_k(\rho(t)) \Pi_k + B_k \Gamma_k(\rho(t)) + E_k(\rho(t)) = \Pi_k \mathbf{S},
\]

\[
G_k(\rho(t)) \Pi_k + R_k \Gamma_k(\rho(t)) + T_k(\rho(t)) = \mathbf{0},
\]

(33)

for any $k \in \mathcal{N}$. We consider that $\Gamma_k(\rho(t))$ has an affine structure as in (31), i.e. $\Gamma_k(\rho(t)) = \Gamma_k + \sum_{i=1}^{p_k} \rho_i(t) \Gamma_i$, for $k \in \mathcal{N}$. The following equations are obtained by substituting (31) into (33)

\[
\begin{bmatrix}
A_i \Pi_k + B_i \Gamma_i + E_i - \Pi_k S \\
G_i \Pi_k + R_i \Gamma_i + T_i = 0
\end{bmatrix}
\begin{bmatrix}
I_{n_x} \\
\vdots \\
I_{n_x}
\end{bmatrix}

+ \sum_{i=1}^{p_k} \rho_i(t) \begin{bmatrix}
A_i \Pi_k + B_i \Gamma_i + E_i \\
G_i \Pi_k + R_i \Gamma_i + T_i
\end{bmatrix} = \mathbf{0},
\]

and hence

\[
A_k \Pi_k + B_k \Gamma_k + E_k - \Pi_k S = 0,
\]

\[
G_k \Pi_k + R_k \Gamma_k + T_k = 0
\]

(34)

for any $k \in \mathcal{N}$ and $i \in \{1, ..., p_k\}$. These equations can be rewritten in the following forms by applying the Kronecker product notation and the vectorisation operator:

\[
(\Pi_k \otimes \mathbf{I}) \hat{\mathbf{y}}_k = - \mathbf{S} \mathbf{y}_k,
\]

(35)

for any $k \in \mathcal{N}$ and $i \in \{1, ..., p_k\}$. Finally, for any $k \in \mathcal{N}$, (32) is obtained by combining the $\mathbf{p}_k$ in (35).

Lemma 5 provides a simple condition to guarantee the solvability of (32).

**Lemma 5:** Consider that Assumptions 1–3 are satisfied. Then, (32) has a solution if

\[
n_{v_k} + \frac{p_k}{p_k + 1} n_{v_k} \leq n_{v_k}.
\]

(36)

for any $k \in \mathcal{N}$.

**Proof:** The number of equations and the number of free variables in linear equation (32) are $n_{v_k}(p_k + 1)(n_{v_k} + n_{x_k})$ and $n_{v_k}(n_{x_k} + (p_k + 1)n_{x_k})$, respectively. The block rows of (32) are linearly independent. If inequality $n_{v_k}(p_k + 1)(n_{v_k} + n_{x_k}) \leq n_{v_k}(n_{v_k} + (p_k + 1)n_{x_k})$ is satisfied, then the system of linear equations (32) has a solution. The latter condition results in the inequality (36).

**Remark 3:** In the case of LTI agents, where $p_k = 0$ for $k \in \mathcal{N}$, inequality (36) becomes $n_{v_k} \leq n_{v_k}$, which is independent of $n_{x_k}$. This has been shown in [1, 22, 23].

4 Illustrative examples and discussion

In this section, two illustrative examples are given to demonstrate the efficacy of the proposed cooperative control design methods of this paper.

**Example 1:** Consider a group of four agents modelled as second-order LPV systems affinely dependent on three scheduling variables. The adjacency matrix of the graph between the agents is assumed to be

\[
\mathbf{A} = \begin{bmatrix}
0 & 1 & 0 & 0 \\
0 & 0 & 0 & 0 \\
0 & 1 & 0 & 0 \\
1 & 1 & 1 & 1
\end{bmatrix}
\]

The state-space matrices of the agents and the exosystem are generated randomly in MATLAB and are as follows:

\[
A_i = \begin{bmatrix}
-3.3865 & -0.3845 \\
0.0599 & -3.0343 \\
-2.9233 & 0.1161 \\
-0.2552 & -3.3113
\end{bmatrix},
\]

\[
A_i = \begin{bmatrix}
-3.2984 & 0.2357 \\
-0.4126 & -4.1131 \\
-3.9176 & -0.0999 \\
1.0752 & -3.2841
\end{bmatrix},
\]
Fig. 2 Directed communication graph for the illustrative example taken from [16, 23]

\[
A_i = \begin{bmatrix}
-3.8459 & -1.4669 \\
0.0003 & -3.0171
\end{bmatrix},
A_i' = \begin{bmatrix}
-3.3934 & 0.0497 \\
-0.1699 & -3.5838
\end{bmatrix},
A_i = \begin{bmatrix}
-4.2923 & -2.4168 \\
0.1082 & -3.1469
\end{bmatrix},
A_i' = \begin{bmatrix}
-3.8434 & -0.5823 \\
0.4239 & -2.7509
\end{bmatrix},
A_i = \begin{bmatrix}
-3.1398 & -0.0405 \\
0.0149 & -3.0884
\end{bmatrix},
A_i' = \begin{bmatrix}
-4.5387 & -0.4815 \\
1.9548 & -2.5680
\end{bmatrix},
A_i = \begin{bmatrix}
-3.6642 & 0.4051 \\
-0.1073 & -4.1063
\end{bmatrix},
A_i' = \begin{bmatrix}
-3.8502 & -0.2520 \\
0.7825 & -2.8082
\end{bmatrix},
A_i = \begin{bmatrix}
-3.5044 & 0.4246 \\
-0.1447 & -4.0079
\end{bmatrix},
A_i' = \begin{bmatrix}
-1.8157 & 2.1903 \\
-0.8780 & -4.6062
\end{bmatrix},
A_i = \begin{bmatrix}
-3.5530 & 0.0002 \\
0.0004 & -3.5537
\end{bmatrix},
A_i' = \begin{bmatrix}
-3.8983 & -2.8897 \\
0.1725 & -2.4550
\end{bmatrix},
C_i = G_i = [0.8275 \ 0],
C_i = G_i = [0.8205 \ 0],
C_i = G_i = [0.8275 \ 0],
C_i = G_i = [0.1641 \ 0],
C_i = G_i = [0.1592 \ 0],
C_i = G_i = [0.2436 \ 0],
C_i = G_i = [0.3981 \ 0],
C_i = G_i = [0.4456 \ 0],
C_i = G_i = [0.4822 \ 0],
C_i = G_i = [0.7549 \ 0],
C_i = G_i = [0.9212 \ 0],
C_i = G_i = [0.9648 \ 0],
C_i = G_i = [0.6584 \ 0],
C_i = G_i = [0.7225 \ 0],
C_i = G_i = [0.8642 \ 0].
\]

\[
E_i = \begin{bmatrix}
1.1725 & 0.4196 \\
0.0183 & 2.0506
\end{bmatrix},
E_i' = \begin{bmatrix}
1.0121 & -0.0402 \\
0.3194 & 2.1464
\end{bmatrix},
E_i = \begin{bmatrix}
1.1420 & -0.0816 \\
1.0825 & 2.4241
\end{bmatrix},
E_i' = \begin{bmatrix}
1.3564 & 0.0034 \\
0.4678 & 2.1370
\end{bmatrix},
E_i = \begin{bmatrix}
3.5102 & 2.0940 \\
-0.5514 & 3.7537
\end{bmatrix},
E_i' = \begin{bmatrix}
3.0972 & -0.4554 \\
0.9736 & 4.2709
\end{bmatrix},
E_i = \begin{bmatrix}
3.9175 & 2.2058 \\
0.6027 & 3.8722
\end{bmatrix},
E_i' = \begin{bmatrix}
3.5078 & 0.5315 \\
0.3564 & 3.5108
\end{bmatrix},
E_i = \begin{bmatrix}
0.5566 & 0.1539 \\
0.5107 & 0.5474
\end{bmatrix},
E_i' = \begin{bmatrix}
0.8045 & 0.0854 \\
0.5173 & 0.4552
\end{bmatrix},
E_i = \begin{bmatrix}
0.6495 & -0.0718 \\
0.7590 & 0.7279
\end{bmatrix},
E_i' = \begin{bmatrix}
0.6825 & 0.0447 \\
0.5126 & 0.4978
\end{bmatrix},
E_i = \begin{bmatrix}
0.2885 & 0.0332 \\
-0.0300 & 0.3300
\end{bmatrix},
E_i' = \begin{bmatrix}
0.1495 & -0.1803 \\
0.6186 & 0.3792
\end{bmatrix},
E_i = \begin{bmatrix}
0.2925 & -0.0000 \\
0.7893 & 0.2926
\end{bmatrix},
E_i' = \begin{bmatrix}
0.3210 & 0.2379 \\
0.5944 & 0.2021
\end{bmatrix},
F_0 = [0.2105 \ 0.6219],
F_1 = [0.4253 \ 0.7977],
F_2 = [0.7655 \ 0.3708],
F_3 = [0.4188 \ 0.4240],
F_4 = [0.7346 \ 0.8366],
F_5 = [0.2988 \ 0.7895],
F_6 = [0.5979 \ 0.9816],
F_7 = [0.0763 \ 0.6697].
\]

\[
T_i = \begin{bmatrix}
-0.2865 \\
-0.0054 \\
-0.1498 \\
-0.2223 \\
-0.0794 \\
-0.0595
\end{bmatrix},
T_i = \begin{bmatrix}
0.9127 \\
0.2300 \\
0.7893 \\
0.2925 \\
0.3210 \\
0.5944
\end{bmatrix},
S = \begin{bmatrix}
0.7225 & 0 \\
0.6584 & 0 \\
0.7549 & 0 \\
0.1592 & 0 \\
0.8275 & 0
\end{bmatrix},
\]

for \(i \in \{0, 1, 2, 3\}\). In addition, following trajectories are considered for the scheduling variables:

\[
\rho_i(t) = a_i(1 - \sin(\alpha_i(t))).
\]

for \(k \in \{1, 2, 3, 4\}\) and \(j \in \{1, 2, 3\}\), where \(0 \leq \alpha_i \leq 1\) and \(0 \leq \alpha_j \leq 1\) are randomly generated numbers. According to the given matrices \(F_i\), agents 1 and 2 are considered to be informed while the uninformod group contains agents 3 and 4. Fig. 2 illustrates the network topology of the agents and the exosystem borrowed from [16, 23]. Agents 1 and 2 in the informed group estimate the states of their associated systems and the exogenous signal. Then, they share the estimation of the exogenous signal with agents 3 and 4 in the uninformod group.

Matrices \(K_i^+\), \(K_i^-\), \(L_i^+\), \(L_i^-\) and \(L_i^w\) are determined by, respectively, solving the linear matrix inequity (LMI) problems in (7), (12), (8) and (13) to ensure that Assumptions 1, 3 and 4 are satisfied. These matrices are determined to

\[
K_i^+ = \text{diag}([-0.1437 \ 0.7606], [0.2537 \ 0.1916]),
K_i^- = \text{diag}([-1.6194 \ 0.3041], [-0.9235 \ -1.6118]),
L_i^+ = \begin{bmatrix}
120.5044 & -273.1877 \\
-271.2264 & 31.2919
\end{bmatrix},
L_i^- = \begin{bmatrix}
-599.4185 & -45.5878 \\
1.0751 & 0.0700
\end{bmatrix},
L_i^w = \begin{bmatrix}
-1.0558 & -0.7275 \\
-0.2865 & -0.9127
\end{bmatrix},
\]

Solving (23) and (26) at the corners (vertices) of the polytope, formed by the extreme values of the scheduling variables, lead to the following solutions:

\[
\Pi^+ = \begin{bmatrix}
0.3462 & 0 \\
0 & 0.3462
\end{bmatrix},
\Pi^- = \begin{bmatrix}
0.9127 & 0 \\
0 & 0.9127
\end{bmatrix}.
\]
whose state-space matrices are calculated using (22) and (25) with models borrowed from [28] are as follows:

Fig. 3 Regulated outputs for the four agents of the given numerical example

Fig. 4 Regulated outputs for the four UAVs

\[ A_i^0 = \begin{bmatrix} -0.4726 & -17.6449 \\ 17.6449 & -0.4726 \end{bmatrix}, \]

\[ A_i^1 = \begin{bmatrix} 4.25 \times 10^{-3} & 1.05 \times 10^{-3} \\ -1.05 \times 10^{-3} & 4.25 \times 10^{-3} \end{bmatrix}, \]

\[ C_i^0 = \begin{bmatrix} 0.0606 & 0.5817 \end{bmatrix}, \quad C_i^1 = \begin{bmatrix} 0.4121 & 0.5004 \end{bmatrix}, \]

\[ G_i^0 = \begin{bmatrix} 0.3742 & 0.3591 \end{bmatrix}, \quad G_i^1 = \begin{bmatrix} 0.5548 & 0.4262 \end{bmatrix}, \]

\[ E_i^0 = \begin{bmatrix} 0.7504 & 6.8798 \\ -5.5827 & 0.6562 \end{bmatrix}, \quad E_i^1 = \begin{bmatrix} 0.5018 & 0.9997 \\ 0.2513 & 0.5000 \end{bmatrix}, \]

\[ F_i^0 = \begin{bmatrix} -0.0200 & -0.1923 \end{bmatrix}, \quad F_i^1 = \begin{bmatrix} -0.1362 & -0.1654 \end{bmatrix}, \]

\[ T_i^0 = \begin{bmatrix} 0.7174 & 0.6979 \end{bmatrix}, \quad T_i^1 = \begin{bmatrix} 0.7937 & 0.4182 \end{bmatrix}, \]

\[ B_i^0 = \begin{bmatrix} 0.5 \end{bmatrix}, \quad D_i^0 = 0, \quad R_i^0 = 0, \quad S = \begin{bmatrix} 0 & 0.795 \\ -0.795 & 0 \end{bmatrix}. \]

for \( k \in \{1, 2, 3, 4\} \) and \( i \in \{0, 1\} \). Similar to Example 1, a sinusoidal function is considered as the scheduling variable, and the adjacency matrix of the graph between UAVs is considered to be the same as in Example 1. UAVs 1 and 2 are considered to be informed while the uninformed group contains UAVs 3 and 4, as shown in Fig. 2. Matrices \( K_i^0, K_i^1, L_i^0, L_i^1 \) and \( L_i^w \) are, respectively, determined by solving the LMI problems in (7), (12), (8) and (13). These matrices are obtained as

\[ K_i^0 = \text{diag}([-0.0095 & -0.0047], [-0.0095 & -0.0047]), \]

\[ K_i^1 = \text{diag}([-0.0095 & -0.0047], [-0.0095 & -0.0047]). \]

\[ L_i^0 = \begin{bmatrix} 0.3002 & 0.5083 \\ 0.2365 & 0.4106 \end{bmatrix}, \quad L_i^1 = \begin{bmatrix} 0.1861 & 0.2171 \\ 0.7773 & 0.7224 \end{bmatrix}, \]

\[ L_i^w = \begin{bmatrix} 0.3002 & 0.5083 \\ 0.2365 & 0.4106 \end{bmatrix}. \]

Objectives 1 and 2 are satisfied by the proposed controller (30), whose state-space matrices are calculated using (22) and (25). The tracking errors for the closed-loop system using the proposed controller are shown in Fig. 4.

5 Conclusion

In this paper, the cooperative output regulation problem is investigated for heterogeneous multi-agent LPV systems. The connection among the agents is specified by a directed graph which consists of a directed spanning tree rooted at the exosystem. The problem under study is viewed as an extension of the leader-follower consensus problem for the multi-agent LPV systems. Followed by the design of a distributed observer, an output feedback control law is proposed to solve the LPV control design problem. The control law requires solving a time-varying Sylvester equation, the offline solution to which is obtained assuming that the dynamics of agents are described by affine LPV models. The future prospect of this work is to address the case of existing switching or random communications due to their practicality for many applications of multi-agent systems.

6 References


IET Control Theory Appl., 2017, Vol. 11 Iss. 9, pp. 1394-1403

© The Institution of Engineering and Technology 2017