A Robust MPC for Input-Output LPV Models

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***Abstract*—In this note, a discrete-time robust model predictive control (MPC) design approach is proposed to control systems described by linear parameter-varying models in input-output form subject to constraints. To ensure the stability of the closed-loop system, a quadratic terminal cost along with an ellipsoidal terminal constraint are included in the control optimization problem. The MPC design problem is formulated as a linear matrix inequality problem. The proposed MPC scheme is applied on a continuously stirred tank reactor as an illustrative example.**

***Index Terms*—Linear matrix inequalities, linear parameter-varying systems, predictive control, robustness, stability.**

I. Introduction

Identifying *linear parameter-varying* (LPV) models in *input-output* (IO) form [1] from data has become well supported with several powerful identification approaches and successful applications, e.g., [2]. However, most of the LPV controller synthesis techniques have been developed for *state-space* (SS) models, e.g., [3]. Obtaining reliable SS realization of IO models is usually hindered by the so-called dynamic-dependency problem connected to LPV realization theory [1], which introduces a significant complexity increase of the realized models that grows beyond the applicable range of computational tools. On the other hand, direct identification of LPV-SS models is still in an immature state either effected by serious approximations of the data equations or computability problems for real-world applications. Therefore, it is desired to synthesize controllers using LPV-IO models directly.

Model predictive control (MPC) has been developed to solve control problems that have constraints and time delay. In the SS setting, the MPC problem has received considerable research interest also addressing the stability issue, e.g., [4], resulting in several different stabilizing MPC schemes. LPV systems have been also investigated in the MPC community and various techniques have been developed. The control law in most of theses techniques, e.g., [5], is calculated by repeatedly solving a convex optimization problem based on *linear matrix inequalities* (LMIs) to minimize a worst-case upper bound of the cost function involving stability constraints. A common property of most introduced MPC techniques based on LPV-SS models is that they rely on the availability of the system states during control implementation. Besides the above mentioned realization problem and immaturity of LPV-SS identification, the use of observers to estimate the states may also deteriorate closed-loop performance significantly in terms of input disturbance rejection when input constraints become activated [6]. To handle this, a subspace-based predictive control for LPV systems has been proposed in [7] without stability guarantee. However, the complexity of this scheme increases exponentially with the order and number of scheduling variables.

To cope with the above issues, we develop in this work a robust MPC approach, which guarantees closed-loop asymptotic stability, to control LPV-IO models subject to constraints. For the sake of simplicity, we focus on the SISO case. To ensure stability, we utilize the stability framework of [4]. The proposed MPC design approach is formulated as an optimization problem subject to LMI constraints. Employing full-block multipliers [8] turns the MPC problem into an optimization problem subject to a finite number of LMI constraints. The significance of the proposed control approach lies in the fact that it enables MPC control design directly based on LPV-IO representations with constraints. In addition, it offers tracking a given reference signal with integral action and asymptotic stability guarantee, for which only past values of the system output and input are required during implementation. The applicability of the approach is demonstrated on a simulation study of a *continuously stirred tank reactor* (CSTR).

Notations: For a sequence $z(k):\mathbb{Z}\to\mathbb{R}$, let $z_{[k+i,k+j]}\in\mathbb{R}^{|i-j|+1}$ gather the values of z ordered from the sampling instant $k+i$ to $k+j$, $i,j\in\mathbb{Z}$. For a matrix $Z\in\mathbb{R}^{n\times m}$, let $Z_{i,j}\in\mathbb{R}^{(j-i+1)\times m}$ gather the rows of Z ordered from row i to j. An upper linear fractional transformation (LFT) is denoted by $\Delta\star\left[\begin{array}{c|c}L_{11} & L_{12}\\ \hline L_{21} & L_{22}\end{array}\right]=L_{22}+L_{21}\Delta(I-L_{11}\Delta)^{-1}L_{12}$.

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II. Preliminaries

An input-output representation of a SISO LPV system in discrete time can be given by the difference equation

$$\mathcal{G}:\left(1+\sum_{i=1}^{n_a}a_i(p_k)q^{-i}\right)y(k)=\sum_{j=0}^{n_b}b_j(p_k)q^{-j}u(k)\tag{1}$$

or $\mathcal{A}(q^{-1},p_k)y(k)=\mathcal{B}(q^{-1},p_k)u(k)$, where q^{-1} is the backward time-shift operator, $n_a,n_b\geq 0$, $u(k):\mathbb{Z}\to\mathbb{R}$ and $y(k):\mathbb{Z}\to\mathbb{R}$ are the control input and the measured output, respectively. Furthermore, the coefficients a_i and b_j are analytic and bounded functions of the scheduling variable $p_k=p(k)\in\mathbb{P}$, which is online measurable. For simplicity, we consider $b_0(p_k)\equiv 0$. Assume that $\mathbb{P}$ is given by a convex set $\mathbb{P}:=\mathrm{Co}(\{p_1^v,\ldots,p_{n_v}^v\})$, where each $p_i^v\in\mathbb{R}^{n_p}$ corresponds to a vertex of a polytope and Co denotes the convex hull. Moreover, let the rate of variation of the scheduling variable $dp(k)=p(k)-p(k-1)$ be bounded such that $dp(k)\in\mathbb{P}_{\mathrm{d}}:=\{dp\in\mathbb{R}^{n_p}\mid dp_{\min}\leq dp\leq dp_{\max}\}$. Consider the reference tracking problem depicted in Fig. 1 and assume that there exists a robust, *linear time-invariant* (LTI)

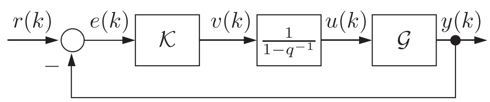

Fig. 1. Closed-loop interconnection: reference tracking.

controller $\mathcal{K}$ which can stabilize the depicted closed-loop system for all $p \in \mathbb{P}$. $\mathcal{K}$ with the integral action can be written in an IO form as

$$\mathcal{K}_{\mathrm{I}}: \left(1+\sum_{i=1}^{n_{\mathrm{Ka}}} a_{\mathrm{K}i} q^{-i}\right)(1-q^{-1}) u(k)=\sum_{j=0}^{n_{\mathrm{Kb}}} b_{\mathrm{K}j} q^{-j} e(k) \tag{2}$$

or $\mathcal{A}_{\mathrm{KI}}(q^{-1})u(k) = \mathcal{B}_{\mathrm{K}}(q^{-1})e(k)$, where $e(k) = r(k) - y(k)$; let $\mathcal{A}_{\mathrm{KI}}(q^{-1}) = 1 + \sum_{i=1}^{n_{\mathrm{Ka}}+1} a_{\mathrm{KI}i} q^{-i}$ and $b_{\mathrm{K}0} = 0$. The closed-loop behavior of the system shown in Fig. 1 can be described implicitly in a so-called LPV kernel representation as [1]

$$\begin{bmatrix} \bar{A}(p_k) & -\bar{B}(p_k) & 0 \\ \bar{B}_{\mathrm{K}} & \bar{A}_{\mathrm{KI}} & -\bar{B}_{\mathrm{K}} \end{bmatrix} \zeta(k) = D(p_k)\zeta(k) = 0 \tag{3}$$

where $\bar{A} = [1\ \ a_1 \ldots a_{n_{\mathrm{dy}}}]$, $\bar{B} = [0\ \ b_1 \ldots b_{n_{\mathrm{du}}}]$ are matrix valued functions, $\bar{A}_{\mathrm{KI}} = [1\ \ a_{\mathrm{KI}1} \ldots a_{\mathrm{KI}n_{\mathrm{du}}}]$, $\bar{B}_{\mathrm{K}} = [0\ \ b_{\mathrm{K}1} \ldots b_{\mathrm{K}n_{\mathrm{dy}}}]$ with $n_{\mathrm{dy}} = \max(n_a, n_{\mathrm{Kb}})$ and $n_{\mathrm{du}} = \max(n_{\mathrm{b}}, n_{\mathrm{Ka}}+1)$ and $\zeta(k) = [y_{[k,k-n_{\mathrm{dy}}]}^{\top}\ u_{[k,k-n_{\mathrm{du}}]}^{\top}\ r_{[k,k-n_{\mathrm{dy}}]}^{\top}]^{\top} \in \mathbb{R}^{n_\zeta}$ with $n_\zeta = 2n_{\mathrm{dy}} + n_{\mathrm{du}} + 3$. Based on the choice of a latent variable $x(k) = \Pi_1 \zeta(k)$ with dimension $n_x = 2n_{\mathrm{dy}} + n_{\mathrm{du}}$ for the closed-loop system (3), where $\Pi_1 = \mathrm{diag}(\Pi_{1y}, \Pi_{1u}, \Pi_{1y})$, $\Pi_{1y} = [0 \quad I_{n_{\mathrm{dy}}}]$, $\Pi_{1u} = [0 \quad I_{n_{\mathrm{du}}}]$, it holds that $x(k+1) = \Pi_2 \zeta(k)$, where, $\Pi_2 = \mathrm{diag}(\Pi_{2y}, \Pi_{2u}, \Pi_{2y})$, $\Pi_{2y} = [I_{n_{\mathrm{dy}}} \quad 0]$, $\Pi_{2u} = [I_{n_{\mathrm{du}}} \quad 0]$ with $\Pi_{\mathrm{i}y} \in \mathbb{R}^{n_{\mathrm{dy}} \times (n_{\mathrm{dy}}+1)}$ and $\Pi_{\mathrm{i}u} \in \mathbb{R}^{n_{\mathrm{du}} \times (n_{\mathrm{du}}+1)}$, $i = 1, 2$. Then, a sufficient condition for asymptotic stability of the closed-loop system can be derived as shown in [9], consequently, the controller can be designed.

III. LPV-MPC Scheme

Next, the proposed MPC technique is developed. Temporarily, assume that the future trajectory of p over the prediction horizon is available. First, the prediction equation used for the MPC formulation is established to express prediction of the future output sequence based on the past measurements generated by model (1). The LPV system represented by (1) has an *infinite impulse response* (IIR) representation in the form of $y(k) = \sum_{i=0}^{\infty} h_i(p_{[k,k-i]})u(k-i)$, where $h_i(\cdot)$ are the Markov coefficients of the LPV system. For simplicity of the notation, we use the short form $h_i(k) = h_i(p_{[k,k-i]})$. Based on (1), the Markov coefficients can be computed recursively

$$h_i(k) = \begin{cases} b_i(p_k) - \displaystyle\sum_{j=1}^{\min(i,n_a)} a_j(p_k) h_{i-j}(k-j), & i \le n_b \\ -\displaystyle\sum_{j=1}^{\min(i,n_a)} a_j(p_k) h_{i-j}(k-j), & \text{else.} \end{cases}$$

In case of no additional disturbances, for given $p_{[k,k+N]}$ and $u_{[k,k+N-1]}$, the future output of $\mathcal{G}$ can be computed as $y(k+j) = \theta^{\top}(k+j)\phi(k) + \sum_{i=1}^{j} h_i(k+j)u(k+j-i)$, $j = 1, 2, \ldots, N$, where N is the prediction horizon, $\phi(k) \in \mathbb{R}^{n_a+n_b}$ is the regressor vector given as $\phi(k) = [y(k-1)\ \ldots\ y(k-n_a)\ u(k-1)\ \ldots\ u(k-n_{\mathrm{b}})]^{\top}$, and $\theta(k+j) \in \mathbb{R}^{n_a+n_b}$ is computed by

$$\theta(k+j) = -\sum_{i=1}^{\min(j,n_a)} a_i\left(p(k+j)\right)\theta(k+j-i) + \overrightarrow{I^j}\bar{\theta}(k+j) \tag{4}$$

$j = 1, 2, \ldots, N$, with $\bar{\theta}(k+j) = [-a_1(p(k+j))\ \ldots\ -a_{n_a}(p(k+j))b_1(p(k+j))\ \ldots\ b_{n_b}(p(k+j))]^{\top}$ and $\overrightarrow{I^j} = \mathrm{diag}(\overrightarrow{I_{n_a}^j}, \overrightarrow{I_{n_b}^j})$, where $\overrightarrow{I_{n_a}^j} \in \mathbb{R}^{n_a \times n_a}$, $\overrightarrow{I_{n_b}^j} \in \mathbb{R}^{n_b \times n_b}$ are obtained by shifting identity matrices of the corresponding dimensions j columns to the right. In order to provide a controller with an integral action, an incremental IO model can be defined by introducing a new input signal as $v(k) = u(k) - u(k-1)$. This yields zero steady-state tracking error under the assumption that that the model gives unbiased steady-state prediction even in the presence of modeling error, disturbances or noise. Therefore, the LPV model can be rewritten as

$$\mathcal{G}_{\mathrm{I}}: \mathcal{A}(q^{-1}, p_k)y(k) = \mathcal{B}(q^{-1}, p_k)\left(v(k) + u(k-1)\right). \tag{5}$$

Now, for given $p_{[k,k+N]}$ and $v_{[k,k+N-1]}$, the future output of $\mathcal{G}_{\mathrm{I}}$ in (5) can be computed as $y(k+j) = \tilde{\theta}^{\top}(k+j)\phi(k) + \sum_{i=1}^{j}\sum_{l=1}^{i} h_l(k+j)v(k+j-i)$, $j = 1, 2, \ldots, N$, where the vector $\tilde{\theta}(k+j) \in \mathbb{R}^{n_a+n_b}$ is computed as in (4) except for its (n_a+1)th element that is given by $\tilde{\theta}_{n_a+1}(k+j) = \theta_{n_a+1}(k+j) + \sum_{i=1}^{j} h_i(k+j)$. Therefore, the key prediction equation for $\mathcal{G}_{\mathrm{I}}$ with $h_0(k) \equiv 0$ can be given by

$$y_{[k+1,k+N]} = H(k)v_{[k,k+N-1]} + \Theta(k)\phi(k) \tag{6}$$

where $H(k) \in \mathbb{R}^{N \times N}$ is a lower triangular Toeplitz matrix with the Markov coefficients of the system

$$H(k) = \begin{bmatrix} h_1(k+1) & \ldots & 0 \\ \vdots & \ddots & \vdots \\ \sum_1^N h_i(k+N) & \ldots & h_1(k+N) \end{bmatrix}$$

$$\Theta(k) = \begin{bmatrix} \tilde{\theta}(k+1) \\ \tilde{\theta}(k+2) \\ \vdots \\ \tilde{\theta}(k+N) \end{bmatrix} \tag{7}$$

$\Theta(k) \in \mathbb{R}^{N \times (n_a+n_b)}$; H and Θ are functions of $p_{[k,k+N]}$.

Next, the proposed LPV-MPC scheme with stability guarantees is formulated. Define the cost function

$$V_N = \sum_{i=0}^{N_e} \underbrace{\mu_i e^2(k+i-n_{\mathrm{dy}})}_{\ell_e(i)} + \sum_{j=1}^{N_v} \underbrace{\rho_j v^2(k+j-n_{\mathrm{du}})}_{\ell_v(j)} + V_f \tag{8}$$

where $V_N = V_N(x_0, e_0, v_{[k,k+N-1]}, r_{[k+1,k+N]}, p_{[k,k+N]})$ with $x_0 \in \mathbb{R}^{n_x}$ is the state vector at instant k and $e_0 = e(k) = r(k) - y(k)$ is the error signal as shown in Fig. 1, $V_f = V_f(x(k+N+1))$ defines a terminal cost with $x(k+N+1) = [y_{[k+N,k+N+1-n_{\mathrm{dy}}]}^{\top}\ u_{[k+N,k+N+1-n_{\mathrm{du}}]}^{\top}\ r_{[k+N,k+N+1-n_{\mathrm{dy}}]}^{\top}]^{\top}$ (to simplify the notation, we drop the arguments) and $N_e = N + n_{\mathrm{dy}} - 1$, $N_v = N + n_{\mathrm{du}} - 1$, $N_v \le N_e$. The terminal cost penalizes the states of the closed-loop system at the end of the prediction horizon, whereas the stage cost given by $\ell(e, v) = \ell_{\mathrm{e}}(i) + \ell_v(j)$ specifies the desired control performance via arbitrary values for $N, \mu_i > 0$ and $\rho_j > 0$, see (8). The proposed MPC control problem can be defined as follows:

$$\min_{v_{[k,k+N-1]}} \quad V_N \tag{9a}$$

$$\text{subject to} \quad u(k+i) \in \mathbb{U}, \quad i = 0, 1, \ldots, N-1 \tag{9b}$$

$$x(k+N+1) \in \mathbb{X}_f \tag{9c}$$

with the system dynamics given by (5), where $\mathbb{U} := \{u(k) \in \mathbb{R} \mid u_{\min} \le u(k) \le u_{\max}\}$ is a compact input set constraint and $\mathbb{X}_f \in \mathbb{R}^{n_x}$ specifies a terminal set constraint to enforce the states at the end of the prediction horizon to lie in $\mathbb{X}_f$.

Introduce $\mathcal{X}_s := \{x_s \in \mathbb{R}^{n_x} \mid x_s = [r_s \ \dots \ r_s \ u_s \ \dots \ u_s \ r_s \ \dots \ r_s]^\top, \ \forall (r_s, u_s) \in \mathcal{R}_s \times \mathcal{U}_s\}$ as the set of all target steady-states x_s, where $\mathcal{R}_s := \{r_s \in \mathbb{R} \mid r_{\min} \leq r_s \leq r_{\max}\}$ and $\mathcal{U}_s := \{u_s \in \mathbb{U} \mid (1 + \sum_{i=1}^{n_a} a_i(p_s)) r_s = (\sum_{j=1}^{n_b} b_j(p_s)) u_s, \ \forall (r_s, p_s) \in \mathcal{R}_s \times \mathbb{P}\}$, define, respectively, the set of all target steady-state references r_s and the set of the corresponding steady-state inputs u_s. Now, consider the following assumptions:

- A.1 There is no model error and no disturbances and the future trajectories of both r and p are known.
- A.2 The reference trajectory r is a piecewise constant signal, and for any target output $y(k+N) = r_s$, $r_s \in \mathcal{R}_s$, all steady-states of the system, i.e., $x(k+N+1) = x_s$ should belong to the terminal set $\mathbb{X}_f$, namely, $\mathcal{X}_s \subset \mathbb{X}_f$.
- A.3 $V_f(\cdot)$ is continuous, positive definite $\forall\, x(k)$ and $V_f(0) = 0$.
- A.4 The set $\mathbb{X}_f$ is closed.

The closed-loop system can be asymptotically stabilized by the MPC control law $\kappa_N(\cdot)$ if there exists a terminal controller $\kappa_f(x(k))$ such that the following conditions are satisfied [4]:

- C.1 $V_f(\cdot)$ is a Lyapunov function on the terminal set $\mathbb{X}_f$ under the controller $\kappa_f(\cdot)$ such that

$$V_f\left(x(k+1)\right) - V_f\left(x(k)\right) \leq -\ell\left(x(k), \kappa_f\left(x(k)\right)\right) < 0 \tag{10}$$

$\forall x(k) \in \mathbb{X}_f, \forall p(k) \in \mathbb{P}, \forall k > N.$

- C.2 The set $\mathbb{X}_f$ is positively invariant under the controller $\kappa_f(\cdot)$, i.e., if $x(k) \in \mathbb{X}_f$, then $x(k+1) \in \mathbb{X}_f, \ \forall\, p \in \mathbb{P}$.
- C.3 $\kappa_f(\cdot) \in \mathbb{U}, \ \forall\, x \in \mathbb{X}_f$.

Under these conditions, the optimal cost function V_N^* is a Lyapunov function for the closed-loop system and its domain of attraction, denoted by $\mathcal{X}_N$, is the set of initial state x_0, initial error e_0 and future reference and scheduling trajectories, $r_{[k+1,k+N]}, p_{[k+1,k+N]}$, respectively, where the optimization problem is feasible. The invariance condition imposed on the terminal region makes the optimization problem feasible if the initial values are in the domain of attraction, c.f., [4].

Next, we show how $V_f(\cdot)$ and $\mathbb{X}_f$ can be chosen to satisfy the above conditions. In terms of (10), the function $V_f(\cdot)$ can be chosen to be an upper bound on the value function of the unconstrained infinite horizon cost of the system states starting from $\mathbb{X}_f$ and controlled by $\kappa_f(\cdot)$ [4]. Thus, we choose

$$V_f\left(x(k{+}N{+}1)\right) \geq \sum_{i=N+1}^{\infty} \left(\tilde{\mu} e^2(k{+}i{-}1) + \tilde{\rho} v^2(k+i-1)\right) \tag{11}$$

for all $x \in \mathbb{X}_f, \ \forall\, p \in \mathbb{P}$ where $\tilde{\mu} > 0$ and $\tilde{\rho} > 0$ are constants. To verify (11), we need to satisfy

$$\begin{aligned} &V_f\left(x(k+i+1)\right) - V_f\left(x(k+i)\right) \\ &\quad \leq -\left(\tilde{\mu} e^2(k+i-1) + \tilde{\rho} v^2(k+i-1)\right) < 0 \end{aligned} \tag{12}$$

for all $e(k+i-1) \neq 0$, $v(k+i-1) \neq 0$, $i \geq N+1$ and $\forall\, p \in \mathbb{P}$. Then, summing (12) from $i = N+1$ to ∞ gives $V_f(x(\infty)) - V_f(x(k+N+1)) \leq -\sum_{i=N+1}^{\infty} (\tilde{\mu} e^2(k+i-1) + \tilde{\rho} v^2(k+i-1))$. If (12) is satisfied, then with Assumption A.3, we have $V_f(x(k+N+1)) \geq V_f(x(\infty))$, and hence, (11) holds. Therefore, Condition C.1 can be verified if there exists a function $V_f(\cdot)$ that satisfies Assumption A.3 along with (12).

Next, we see how (12) can be attained. If there exists a function $V_f(\cdot)$ that satisfies Assumption A.3 and the inequality (12), then it can serve as a Lyapunov function for the closed-loop system shown in Fig. 1. On the other hand, this also implies the existence of a control law $\kappa_f(\cdot)$ that can drive a state in $\mathbb{X}_f$ into a steady-state point $x_s \in \mathbb{X}_f$, i.e., $\lim_{k\to\infty} \|x(k) - x_s\| = 0$. Therefore, we need to derive a controller such that (12) holds for all $i \geq N+1$, and consequently, it guarantees that $x(\infty)$ approaches x_s. In other words, we employ (12) to design the controller $\kappa_f(\cdot)$, the existence of which implies that $V_f(\cdot)$ is a Lyapunov function for the closed-loop system. This suggests that $V_f(\cdot)$ could be a quadratic function as $V_f(x(k)) = x^\top(k) P_f x(k)$, $P_f = P_f^\top \succ 0$, $P_f \in \mathbb{R}^{n_x \times n_x}$. Then, based on such a $V_f(\cdot)$, (12) and the application of the $\mathcal{S}$-procedure and Finsler's Lemma, we obtain the following sufficient condition.

Theorem 1: The closed-loop system described by (3) is asymptotically internally stable and satisfies the $\mathcal{L}_2$-performance constraint $\zeta^\top(k+i) Q \zeta(k+i) \geq 0$, where $Q = \mathrm{diag}(Q_1, Q_2)$, $Q_1 = \mathrm{diag}(-1, 0, \ldots)$, $Q_2 = \mathrm{diag}(\gamma^2, 0, \ldots)$, $\gamma > 0$, if there exist a controller $\kappa_f(\cdot)$, $\tilde{F} \in \mathbb{R}^{n_\xi \times 2}$, and

$$S = \begin{bmatrix} S_1 & 0 & -S_1 \\ 0 & S_2 & 0 \\ -S_1 & 0 & S_1 \end{bmatrix}, \ S_1 = \begin{bmatrix} \tilde{\mu} & 0 \\ 0 & 0 \end{bmatrix}, \ S_2 = \begin{bmatrix} \tilde{\rho} & -\tilde{\rho} & 0 \\ -\tilde{\rho} & \tilde{\rho} & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

$\tilde{\mu} > 0$, $\tilde{\rho} > 0$, where $S_1 \in \mathbb{R}^{n_{\mathrm{dy}} \times n_{\mathrm{dy}}}$, $S_2 \in \mathbb{R}^{n_{\mathrm{du}} \times n_{\mathrm{du}}}$ s.t.

$$P_f = P_f^\top \succ 0 \tag{13a}$$

$$\Pi_2^\top P_f \Pi_2 - \Pi_1^\top (P_f + \tilde{S}) \Pi_1 + Q + \tilde{F} D(p) + D^\top(p) \tilde{F}^\top \prec 0 \tag{13b}$$

hold for all $p \in \mathbb{P}$.

The proof is omitted due to the lack of space; inequalities (13a), (13b) can be solved as a feasibility problem, see [9]. Therefore, existence of the controller $\kappa_f(\cdot)$ satisfying (13a) and (13b) for all $p \in \mathbb{P}$ guarantees that $V_f(x(k))$ is a Lyapunov function satisfying (12), which implies Condition C.1, and we have

$$V_f\left(x(k)\right) = x^\top(k) P_f x(k), \qquad P_f = P_f^\top \succ 0. \tag{14}$$

Next, we verify Conditions C.2 and C.3. For C.2, it is required to specify $\mathbb{X}_f$ to be a positive invariant set with the controller $\kappa_f(\cdot)$ [4]. We consider an ellipsoidal terminal set $\mathbb{X}_f$ that is a sublevel set of $V_f(\cdot)$ as

$$\mathbb{X}_f := \left\{x(k) \in \mathbb{R}^{n_x} \mid x^\top(k) P_f x(k) \leq \alpha\right\}, \quad \alpha > 0. \tag{15}$$

The sublevel constant α in (15) is maximized such that $K_f x \in \mathbb{U}, \ \forall\, x \in \mathbb{X}_f$, to provide the positive invariance property for $\mathbb{X}_f$ with the controller $\kappa_f(\cdot)$, and hence, Condition C.3 can be verified. Hence, stratification of Conditions 2 and 3 can be attained by solving the convex optimization problem

$$\max_{\tilde{\alpha}} \ \tilde{\alpha} \qquad \text{subject to} \ -u_{\max} \leq \tilde{\alpha} \left\|P_f^{-1} K_f\right\|_2 \leq u_{\max} \tag{16}$$

where $\tilde{\alpha} = \sqrt{\alpha}$. Hence, $\mathbb{X}_f$ in (15) can be redefined as

$$\mathbb{X}_f := \left\{x(k) \in \mathbb{R}^{n_x} \mid x^\top(k) P_f x(k) \leq \alpha_m\right\} \tag{17}$$

where α_m is the solution of (16). To ensure the feasibility of the proposed MPC, the terminal set $\mathbb{X}_f$ should also satisfy Assumption A.2. This can be satisfied by verifying that $\mathbb{X}_f \supset \mathcal{X}_s$. To fulfill this, introduce

$$\alpha_s = \max_{x_s \in \mathcal{X}_s} x_s^\top P_f x_s \tag{18}$$

then, if $\alpha_s \leq \alpha_m$, see (17), we can verify that $\mathcal{X}_s \subset \mathbb{X}_f$.

Finally, we summarize the previous results as follows.

Theorem 2: Suppose that Assumptions A.1, A.2, A.3, and A.4 are satisfied and there exists a terminal cost given by (14) such that (13) is satisfied and a terminal set given by (17) such that $\alpha_m \geq \alpha_s$, where α_m and α_s solve (16) and (18), respectively. Then, Conditions C.1, C.2, and C.3 are satisfied. Consequently, the MPC controller derived by solving the problem (9) asymptotically internally stabilizes the system (5) for all x_0, e_0, $r_{[k+1,k+N]}$ and $p_{[k,k+N]}$ in the set $\mathcal{X}_N$.

Next, the MPC problem (9) is represented as an optimization problem with LMI constraints; this is the key step to formulate the robust LPV-MPC scheme in the next section. The cost function (8) can be rewritten as follows:

$$V_N = V_0 + (*)^\top M \left(r_{[k+1,k+N-1]} - y_{[k+1,k+N-1]}\right) + (*)^\top R v_{[k,k+N-1]} + (*)^\top \tilde{P}_f \tilde{x}(k+N+1) \quad (19)$$

where $V_0 = \sum_{i=0}^{n_{\mathrm{dy}}} \mu_i e^2(k+i-n_{\mathrm{dy}}) + \sum_{j=1}^{n_{\mathrm{du}}-1} \rho_j v^2(k+j-n_{\mathrm{du}})$ is a constant term, $M = \mathrm{diag}\{\mu_{n_{\mathrm{dy}}+1}, \mu_{n_{\mathrm{dy}}+2}, \ldots, \mu_{N_e}\} \in \mathbb{R}^{(N-1)\times(N-1)}$, $R = \mathrm{diag}\{\rho_{n_{\mathrm{du}}}, \rho_{n_{\mathrm{du}}+1}, \ldots, \rho_{N_v}\} \in \mathbb{R}^{N\times N}$, $\tilde{x}(k+N+1) = T_x^{-1} x(k+N+1)$ with $T_x \in \mathbb{R}^{n_x \times n_x}$ is a state transformation given by $T_x = \mathrm{diag}(T_{x1}, T_{x2}, T_{x1})$, with $T_{x1} \in \mathbb{R}^{n_{\mathrm{dy}} \times n_{\mathrm{dy}}}$, $T_{x2} \in \mathbb{R}^{n_{\mathrm{du}} \times n_{\mathrm{du}}}$ are anti-diagonal matrices with all nonzero entries equal to one and $\tilde{P}_f = T_x^\top P_f T_x$. Now, given $p_{[k,k+N]}$ and $r_{[k+1,k+N]}$, we can rewrite Problem (9) as

$$\min_{\beta, v_{[k,k+N-1]}} \quad \beta \quad (20a)$$

$$\text{subject to} \quad V_N \le \beta \quad (20b)$$

$$u(k+i) \in \mathbb{U}, \quad i = 0, 1, \ldots, N-1 \quad (20c)$$

$$x(k+N+1) \in \mathbb{X}_f. \quad (20d)$$

Substituting (7) into (20b), and then applying Schur complement leads to an LMI constraint for (20b) as shown in (21), at the bottom of the page, where $1_\chi = [1\,1 \cdots 1]^\top \in \mathbb{R}^\chi$ and $T_u \in \mathbb{R}^{n_{\mathrm{du}} \times N_c}$ is given by

$$T_u = \begin{bmatrix} T_{u1} & T_{u2} \\ 1^\top_{N-n_{\mathrm{du}}+1} & 1^\top_{n_{\mathrm{du}}-1} \end{bmatrix}, \quad \begin{matrix} T_{u1} \in \mathbb{R}^{(n_{\mathrm{du}}-1)\times(N_c - n_{\mathrm{du}}+1)} \\ T_{u2} \in \mathbb{R}^{(n_{\mathrm{du}}-1)\times(n_{\mathrm{du}}-1)} \end{matrix}$$

with T_{u1} being a matrix whose entries are all one and T_{u2} is a lower triangular matrix whose non-zero entries are one. Next, the control input constraint (20c) can be written as

$$E v_{[k,k+N-1]} \preceq c, \; E = \begin{bmatrix} T_e \\ -T_e \end{bmatrix}$$
$$c = \begin{bmatrix} 1_N u_{\max} - 1_N u(k-1) \\ -1_N u_{\min} + 1_N u(k-1) \end{bmatrix} \quad (22)$$

with $T_e \in \mathbb{R}^{N\times N}$ being a lower triangular matrix whose non-zero entries are all one. Finally, the terminal set constraint (20d) using (17) can be written as an LMI constraint as

$$\left[\begin{array}{c|c} \tilde{P}_f^{-1} & \begin{matrix} H_{N+1-n_{\mathrm{dy}},N}(k) v_{[k,k+N-1]} \\ +\Theta_{N+1-n_{\mathrm{dy}},N}(k)\phi(k) \\ T_u v_{[k,k+N-1]} + 1_{n_{\mathrm{du}}} u(k-1) \\ r_{[k+N+1-n_{\mathrm{dy}},k+N]} \end{matrix} \\ \hline *^\top & \alpha_m \end{array}\right] \succeq 0. \quad (23)$$

Therefore, (9) can be presented as an optimization problem with LMI constraints as follows: At any time instant k, given x_0, e_0, $p_{[k,k+N]}$, $r_{[k+1,k+N]}$, $\tilde{P}_f$, α_m and appropriate values for N and the matrices M and R, solve

$$\min_{\beta, v_{[k,k+N-1]}} \quad \beta \qquad \text{subject to} \quad (21), (22), (23). \quad (24)$$

This problem is solved online at each time instant k, where N, M, R are tuning parameters. Also, $\tilde{P}_f$ and α_m should be obtained offline by solving the feasibility problem (13) and the optimization problem (16), respectively.

IV. Robust LPV-MPC Scheme

We propose in this section an MPC scheme based on the above formulation to design a robust MPC for LPV-IO models in which at every sampling instant k the instantaneous value of p is given and its future values, i.e., $p(k+1), p(k+2), \ldots, p(k+N)$, required to compute $H(k)$ and $\Theta(k)$, are uncertain. Therefore, in (19), the worst-case cost over all possible future scheduling values is considered. We then employ the full-block multipliers introduced in [8], to provide an optimization problem with a finite number of LMI constraints. Bounds on the rate of variation of p and on its values will be exploited to verify these LMIs at the vertices of a subset of $\mathbb{P}$ that reduces conservatism of the design. The robust MPC design introduced here is based on the full-block $\mathcal{S}$-procedure, which can be used to convert an uncertain matrix inequality to a finite set of inequalities using full-block multipliers, see [8] for more details.

At a sampling instant k, if each of the constraints (21) and (23) can be represented as a certain quadratic form, the full block $\mathcal{S}$-procedure can be used to transform each of them into a form which enables solving the optimization problem (24) without knowing the required future values of p. The first step is to formulate each of the constraints (21) and (23), respectively, as

$$F^\top(p) W_F(k) F(p) \succeq 0 \quad (25a)$$

$$G^\top(p) W_G(k) G(p) \succeq 0 \quad (25b)$$

where $F(p) \in \mathbb{R}^{n_f \times (2N+n_x)}$, $n_F = 4N + n_x + n_a + n_b + n_{\mathrm{du}} + 1$, and $G(p) \in \mathbb{R}^{n_G \times (n_x+1)}$, $n_G = N + n_x + n_a + n_b + n_{\mathrm{dy}} + n_{\mathrm{du}} + 2$ are matrix valued functions of H and Θ, and $W_F \in \mathbb{R}^{n_F \times n_F}$ and $W_G \in \mathbb{R}^{n_G \times n_G}$ are matrix valued functions of r, v, which are not given due to space restrictions.

As a consequence, (25a) and (25b) can replace (21) and (23), respectively, in the optimization problem (24). Next, we transform both $F(p)$ and $G(p)$ into an upper LFT form as

$$F(p) = \Delta_F \star \left[\begin{array}{c|c} F_{11} & F_{12} \\ \hline F_{21} & F_{22} \end{array}\right], \; G(p) = \Delta_G \star \left[\begin{array}{c|c} G_{11} & G_{12} \\ \hline G_{21} & G_{22} \end{array}\right] \quad (26)$$

$$\left[\begin{array}{ccc|c} M^{-1} & 0 & 0 & \begin{matrix} r_{[k+1,k+N-1]} - H_{1,N-1}(k) v_{[k,k+N-1]} \\ -\Theta_{1,N-1}(k)\phi(k) \end{matrix} \\ 0 & R^{-1} & 0 & v_{[k,k+N-1]} \\ \hline 0 & 0 & \tilde{P}_f^{-1} & \begin{matrix} H_{N+1-n_{\mathrm{dy}},N}(k) v_{[k,k+N-1]} \\ +\Theta_{N+1-n_{\mathrm{dy}},N}(k)\phi(k) \\ T_u v_{[k,k+N-1]} + 1_{n_{\mathrm{du}}} u(k-1) \\ r_{[k+N+1-n_{\mathrm{dy}},k+N]} \end{matrix} \\ \hline *^\top & *^\top & *^\top & \beta - V_0 \end{array}\right] \succeq 0 \quad (21)$$

such that $\Delta_F = \text{diag}\{p_1 I_{r_{F1}}, \ldots, p_{n_p} I_{r_{Fn_p}}\}$, $\Delta_F \in \mathbf{\Delta}_F$, $\Delta_G = \text{diag}\{p_1 I_{r_{G1}}, \ldots, p_{n_p} I_{r_{Gn_p}}\}$, $\Delta_G \in \mathbf{\Delta}_G$, where $\mathbf{\Delta}_F(k) = \{\Delta_F(k) \in \mathbb{R}^{n_{\Delta_F} \times n_{\Delta_F}} | \underline{p}_i(k) \le p_i \le \overline{p}_i(k), i = 1, 2, \ldots, n_p\}$, $\mathbf{\Delta}_G(k) = \{\Delta_G(k) \in \mathbb{R}^{n_{\Delta_G} \times n_{\Delta_G}} | \underline{p}_i(k) \le p_i \le \overline{p}_i(k), i = 1, 2, \ldots, n_p\}$, $n_{\Delta_F} = \sum_{i=1}^{n_p} r_{F_i}$, $n_{\Delta_G} = \sum_{i=1}^{n_p} r_{G_i}$, $\overline{p}_i(k) = \max(N \cdot dp_{\max i} + p_i(k), p_{\min i})$, and $\underline{p}_i(k) = \min(N \cdot dp_{\min i} + p_i(k), p_{\max i})$. Now, if the LFTs (26) are well-posed, then we can apply the results of [8] to the conditions (25a) and (25b). Therefore, at the sampling instant k, given x_0, e_0, $r_{[k+1,k+N]}$, $p(k)$, the parameters $\tilde{P}_f$ and α_m, which can be computed offline, and the design parameters N, M and R, the optimization problem (24) associated with the robust MPC design considered here can be written as in (27a)–(27d), shown at the bottom of the page, $i = 1, 2, \ldots, 2^{n_p}$, where $\Xi_F \in \mathbb{R}^{2n_{\Delta_F} \times 2n_{\Delta_F}}$, $\Xi_G \in \mathbb{R}^{2n_{\Delta_G} \times 2n_{\Delta_G}}$

$$\Xi_F = \begin{bmatrix} \Xi_{F11} & \Xi_{F12} \\ \Xi_{F12}^\top & \Xi_{F22} \end{bmatrix}, \qquad \Xi_G = \begin{bmatrix} \Xi_{G11} & \Xi_{G12} \\ \Xi_{G12}^\top & \Xi_{G22} \end{bmatrix}.$$

Next, as $\mathbb{P}$ is a convex polytope and the blocks Δ_F and Δ_G have linear dependence on p, the LMIs (27c) and (27d) are only required to be solved at the vertices of $\mathbb{P}$, see [8]. Finally, we summarize the proposed robust MPC design as follows.

Theorem 3: Suppose that Assumptions A.1, A.2, A.3, and A.4 are satisfied, and there exists a matrix $P_f = P_f^\top \succ 0$ that satisfies conditions (13) $\forall p \in \mathbb{P}$, and a scalar α_m that solves the problem (16) such that $\alpha_m \ge \alpha_s$, where α_s is a scalar that solves (18). Then, Conditions C.1, C.2, and C.3 are satisfied. Consequently, the robust MPC controller derived by solving the problem (27) stabilizes asymptotically the system (5) for all initial values of x_0, e_0 and $r_{[k+1,k+N]}$ for all time samples greater than the sampling instant k if the problem (27) has a feasible solution at the sampling instant k.

V. Numerical Example

In this section, the performance of the proposed MPC scheme for LPV-IO models is demonstrated using a simulation study. The CSTR system is a challenging chemical process for *nonlinear* (NL) modeling and control [10]. The first-principle based model is given in [10] as

$$\dot{C}_A(t) = q\,(C_{A0}(t) - C_A(t))\,/V - k_0 C_A(t) e^{\frac{-E}{RT(t)}} \tag{28a}$$

$$\dot{T}(t) = q\,(T_0 - T(t))\,/V + \Delta H k_0 C_A(t) e^{\frac{-E}{RT(t)}} / \rho C_p + \rho_c C_{pc} q_c(t) \left(1 - e^{\frac{-hA}{q_c(t)\rho C_p}}\right) (T_{c0} - T(t))\,/\rho C_p V \tag{28b}$$

where C_A is the product concentration in mol/L, T is the temperature of the reactor in K, q_c is the coolant flow rate in L/min, C_{A0}, T_0 are the feed concentration and temperature, respectively, and T_{c0} is the inlet coolant temperature; see [10] for the other parameters. Let C_A and q_c, respectively, be the measured output and the manipulated variable [10]. The considered operating points corresponding to q_c vary from 97.265 to 110.032 L/min, which provide an operating range for C_A between 0.08 and 0.13 mol/L [10]. The variance of the process dynamics over the range of q_c is shown in Fig. 2 (solid line), indicating strong nonlinearity of the process [10]. To implement the proposed MPC scheme, a discrete LPV-IO representation for the NL description (28), in the operating region defined above, is developed using the Jacobian linearization [1]. Therefore, the NL model has been linearized around 20 operating points that provide a set of local LTI-IO models. To achieve appropriate discretization of these models theoreticallya sampling time $T_s^* = 2\pi/20\omega_{\text{bw}} = 2$ sec is needed, where ω_{bw} is the largest control bandwidth required to regulate each of the LTI models. However, it was observed that $T_s = 8$ sec based MPC design using models discretized with T_s could achieve the same performance as T_s^* with less computation time. Next, a polynomial interpolation scheme of the coefficients, see [2], has been applied to construct a global LPV-IO model with the scheduling variable C_A. This provides the LPV-IO model

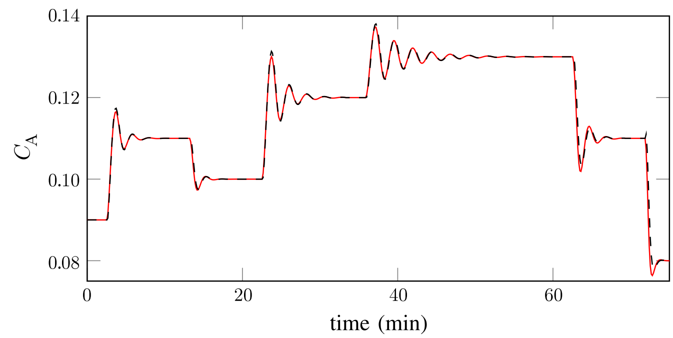

Fig. 2. Open-loop simulation of the CSTR system.

$$C_A(k) = -a_1(p_k) C_A(k-1) - a_2(p_k) C_A(k-2) + b_1(p_k) q_c(k-1) + (1 + a_1(p_k) + a_2(p_k))\, C_{As} - b_1(p_k) q_{cs} \tag{29}$$

where C_{As} and q_{cs} are steady-state values, $p(k) = C_A(k-1)$, $a_1(p_k) = a_{10} + a_{11} p_k + a_{12} p_k^2$, $a_2(p_k) = a_{20} + a_{21} p_k + a_{22} p_k^2$, $b_1(p_k) = b_{10} + b_{11} p_k + b_{12} p_k^2$. In order to remove the offset in the LPV-IO model we introduce a virtual input $\tilde{q}_c$ as follows:

$$\tilde{b}_1 \tilde{q}_c(k-1) = b_1 q_c(k-1) + (1 + a_1 + a_2) C_{As} - b_1 q_{cs} \tag{30}$$

where $\tilde{b}_1$ is a freely chosen constant that can be used to improve numerical properties. To assess the quality of the derived model, Fig. 2 shows the variance of the LPV-IO model dynamics (dashed line) over the specified range of q_c in comparison with that of the NL model. This demonstrates that the LPV model has captured the dynamics of the NL model. The MPC design is performed based on the LPV-IO model

$$C_A(k) = -a_1(p_k) C_A(k-1) - a_2(p_k) C_A(k-2) + \tilde{b}_1 \tilde{q}_c(k-1). \tag{31}$$

$$\min_{\beta, v_{[k,k+N-1]}, \Xi_f, \Xi_G} \quad \beta \tag{27a}$$

$$\text{subject to} \quad E v_{[k,k+N-1]} \preceq c \tag{27b}$$

$$\begin{bmatrix} * \\ * \end{bmatrix}^\top \begin{bmatrix} \Xi_F & 0 \\ 0 & W_F \end{bmatrix} \begin{bmatrix} F_{11} & F_{12} \\ I & 0 \\ F_{21} & F_{22} \end{bmatrix} \succ 0, \quad [*]^\top \Xi_F \begin{bmatrix} I \\ \Delta_{Fi} \end{bmatrix} \prec 0, \qquad \Xi_{F22} \succ 0 \tag{27c}$$

$$\begin{bmatrix} * \\ * \end{bmatrix}^\top \begin{bmatrix} \Xi_G & 0 \\ 0 & W_G \end{bmatrix} \begin{bmatrix} G_{11} & G_{12} \\ I & 0 \\ G_{21} & G_{22} \end{bmatrix} \succ 0, \quad [*]^\top \Xi_G \begin{bmatrix} I \\ \Delta_{Gi} \end{bmatrix} \prec 0, \qquad \Xi_{G22} \succ 0 \tag{27d}$$

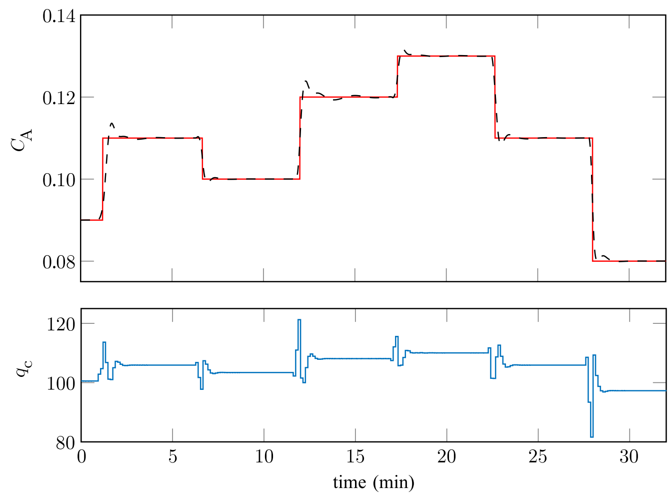

Fig. 3. Closed-loop performance with the proposed robust LPV-MPC scheme: Reference tracking (top); Control input.

Now, at each sampling instant, the MPC algorithm computes $\tilde{q}_c$. Then, the value of q_c is obtained from (30), which in turn is applied to the NL model. Next, the performance of the proposed MPC scheme is demonstrated on simulation for the CSTR NL model (28). The scheduling variable $p = C_A$ is assumed to take values in the range $\mathbb{P} = [0.08,\ 0.13]$. The input constraint is defined as $|\tilde{q}_c| \leq 1.5$ and the reference to be tracked is given in advance as shown in Fig. 3 (solid line). In order to find the terminal cost $V_f(\cdot)$, the feasibility problem defined by (13a), (13b) has been solved to obtain the matrix $P_f \in \mathbb{R}^{6\times 6}$ and the terminal controller $\kappa_f(\cdot)$, with $n_{\mathrm{Ka}} = 1$ and $n_{\mathrm{Kb}} = 2$. Next, the ellipsoidal terminal set $\mathbb{X}_f$ in (17) is constructed offline by computing the value of the parameter α_m by solving Problem (16) and verifying it by solving Problem (18); we obtained $\alpha_m = 0.748$ and $\alpha_s = 0.685$, hence $\alpha_m > \alpha_s$. Given P_f and α_m, which parameterize $V_f(\cdot)$ and $\mathbb{X}_f$, respectively, the proposed MPC schemes can be applied. The tuning parameters have been chosen as $\rho = 300$, $\mu = 3 \times 10^5$, $N = 4$ and $N_c = 3$ to achieve a desired regulation of the CSTR. Then, the robust LPV-MPC scheme has been implemented by solving its associated optimization problem at each sampling instant k to obtain the optimal control law online. To reduce the conservatism, we considered the bounds on the rate of change of p as $|p(k) - p(k-1)| \leq 0.00015$. Based on such bounds and the value of N, a reduced parameter set $\hat{\mathbb{P}}(k) < 0.1 \cdot \mathbb{P}$ can be considered. The resulting control structure has been validated via a simulation study using the original NL model (28). Stability of the closed-loop system over the entire operating region and feasibility of the optimization problem at all sampling instants have been achieved by the MPC design. The evolution of the output and the control input with the MPC controller are shown in Fig. 3. The closed-loop performance of the system with the proposed MPC scheme shows satisfactory tracking capability at different operating conditions and the integral action guarantees zero steady-state tracking error asymptotically. To assess computational complexity, the mean and standard deviation of the CPU time required to solve the optimization problem at each sampling instant was $5.0351 \pm .5947$ s on a 2.13 GHz processor.

Note that an LPV-SS representation for the CSTR system using (28a) and (28b) can be obtained using a set of local LTI-SS models. Unfortunately, interpolation of the state space matrices often leads to unreliable models, and hence, a more computationally demanding behavioral interpolation scheme is advised in the $\mathcal{H}_2/\mathcal{H}_\infty$ sense (see [11]). Then, existing LPV-SS based MPC techniques, e.g., [5], can be utilized to design a controller, which could achieve comparable results as above. Furthermore, T as one of the system states should be measured or estimated to operate the MPC-SS controller, which rises further complications in practice. For the proposed scheme, that is not required. On the other hand, in several chemical processes, like copolymerization process, deriving first principle models is a challenging and often infeasible task; therefore, data-driven techniques appear to be attractive alternatives. Unfortunately, while LPV-IO identification offers powerful tools to estimate models under real word assumption on disturbances and measured noise affecting the captured data, LPV-SS identification is underdeveloped [1]. Existing MPC tools are mostly for SS models and hence, an LPV-SS realization for the identified LPV-IO model should be obtained. As shown in [1], such LPV-SS representations will be more complex than the original LPV-IO model to preserve equivalence. Therefore, the proposed MPC scheme offers a practical solution as it works directly for LPV-IO models.

VI. Conclusion

This note has proposed an MPC design method to control LPV-IO models subject to input constraints with stability guarantee. The proposed LPV-MPC scheme characterizes a robust strategy to counteract the worst-case possible uncertainties of the scheduling variable. To guarantee closed-loop asymptotic stability, an appropriate quadratic terminal cost is added to the quadratic finite horizon cost function of the online MPC optimization problem and an ellipsoidal terminal set constraint is included. A simulation study on a CSTR NL model has demonstrated the applicability of the MPC design scheme.

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