Stochastic Model Predictive Control Method for Microgrid Management

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***Abstract*—This paper presents a stochastic model predictive control method for managing a microgrid. In order to reliably provide the required power for costumers, the proposed method enables the microgrid to use the renewable energy sources as much as possible while keeping the storage device to its maximum state of charge and minimizing the power generated by the micro gas turbine. The performance and effectiveness of the proposed method will be finally illustrated by simulating a microgrid model consisting of three nodes including a renewable generation source and a battery, customers, and a micro gas turbine.**

***Keywords*: Microgrid, renewable energy sources, storage device, stochastic model predictive control, empirical mean, dynamic programming.**

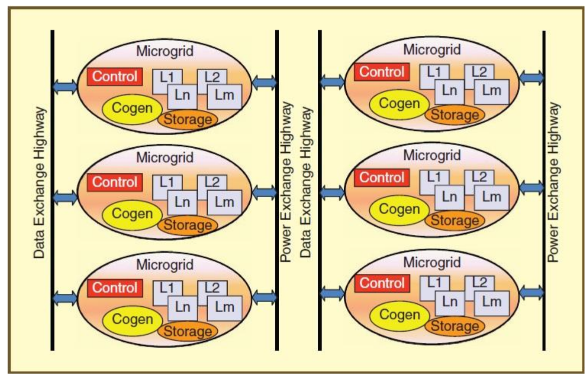

Fig. 1. Smart grid topology [5]

I. Introduction

The future generation of the power networks, or the so-called smart grid [11], is a grid that includes different types of generation options such as central, distributed, and intermittent ones. The future smart grids will create the ability for the consumers to interact with the utility to manage their consumption and minimize the cost of energy [5].

The development and evolution of the smart grids will result in the plug-and-play integration of intelligent structures called microgrids (MG) that will be linked with each other through particular channels for power, information, and control signals exchange [9], [8], [14]. An MG can be a residential place like a house or a commercial area such as a huge shopping center or even a combination of the two, *e.g.,* a city. Figure 1 shows the afore-described topology. As shown in Figure 2, each microgrid by itself can integrate loads and resources such as wind, solar, biomass, and micro gas turbines, as well as storage devices such as battery or electric vehicles.

Using the plug-and-play feature of a microgrid, it can operate in two modes: grid-connected mode or islanded mode, *i.e.,* separated from the grid [7], [12]. In this paper, we mainly focus on the islanded mode operation.

By taking advantage of the concept of MG, the demand management task can be accomplished more efficiently. To this end, there will be a need to an MG central controller (MGCC) to control the operations and perform optimizations so as to minimize the power generation cost. In other words,

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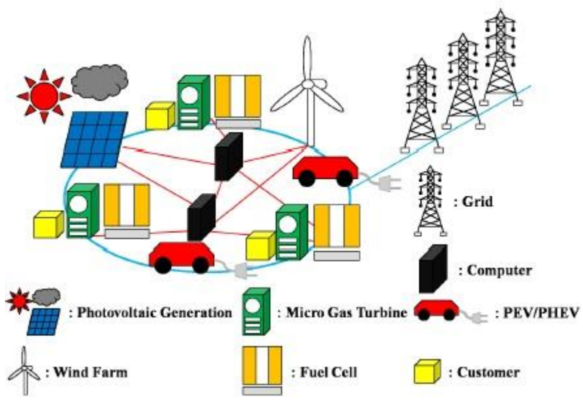

Fig. 2. Microgrid [16]

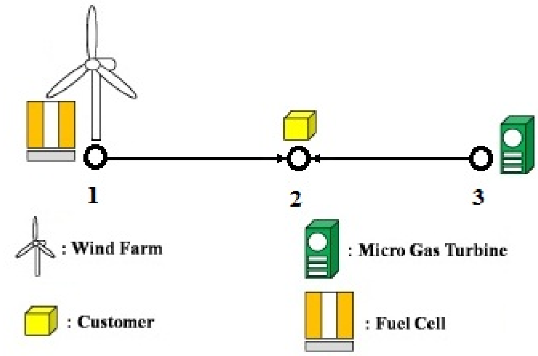

Fig. 3. The microgrid's graph

an MGCC tries to optimize the future behavior of the grid to meet the predicted demand based on forecasted renewable generations. Therefore, due to the nature of this problem, one of the effective tools to achieve the optimized operation is the model predictive control [10]. There are several ways to solve a model predictive control (MPC) problem including dynamic matrix control and dynamic programming. Dynamic matrix control (DMC) method solves the MPC problem by forming an equation in which the future outputs of the system are controlled [19]. The approach that DMC uses for finding the solution is a simple least square method. Although due to the simplicity in the implementation, DMC is favorable in industrial applications like chemical industry, it is hard to deal with stochastic terms in this method. In the problem under study in the present paper, the stochastic terms appear due to the uncertainties in the prediction of demand or renewable energy generations. Moreover, an issue with DMC is the complexity in the sense that when the dimension of the problem increases, the complexity of solution raises as well. The MPC problem can be also solved using dynamic programming (DP) method [1]. The advantage of using DP is that the dimension of the problem does not considerably lead to an increase in the complexity of the solution. More importantly, the stochastic dynamic programming (SDP) method can be employed to enable the DP method to deal with stochastic terms [2]. This method is essentially an extension of Bellman's equations. Solving SDP using stochastic version of Bellman's equations is not, however, easy if there exist state or control input constraints.

Contribution of the present paper is as follows: we first consider a 3-node graph associated with a microgrid with different generation and storage options. To solve the microgrid power management problem taking into account the stochastic disturbance inputs and various constraints imposed by the distribution lines and battery level of charge, we propose a solution method to the stochastic MPC problem motivated by [6] using the empirical mean and dynamic programming tools.

The paper is organized as follows: Section II describes the model of a microgrid and the problem formulation. Stochastic model predictive control policy and algorithm are then described in Section III. Simulation results on the model of a simple microgrid network are then presented in section IV. Section V concludes the paper.

II. System Modeling and Problem Formulation

The microgrid we consider in this paper is assumed to be a graph consisting of three nodes as illustrated in Figure 3. The first node represents the renewable energy generation sources such as a wind turbine and a storage device such as a fuel cell battery. The power generated in this node is:

$$P_1(t) = P^{wind}(t) + w^{wind}(t) + P^{batt}(t) \qquad (1)$$

where $P_1(t)$ is the total power generated in node 1 at time instant t, $P^{wind}(t)$ is the power generated by the wind turbine (or a wind farm in general) at time instant t, and $w^{wind}(t)$ is a disturbance term representing the uncertainty in the wind profile prediction. The latter is assumed to have a gaussian distribution. In addition, $P^{batt}(t)$ is the power generated by the battery. So, the change in battery state of charge (SOC) can be described as [3]:

$$SOC(t+1) = SOC(t) - P^{batt}(t). \quad (2)$$

It is noted that $P^{batt}(t)$ can be either positive or negative, where a negative value implies that the battery is being charged. Due to a limit on the battery level of charge, we have the following constraint for SOC:

$$0 \leq SOC(t) \leq SOC^{max}. \quad (3)$$

Node 2 in Figure 3 represents customers connected through the transmission line to Node 1. Hence, the total generated power $P_1(t)$ should not exceed the line capacity. This constraint can be mathematically represented by:

$$0 \leq P_1(t) \leq L_{12}^{max} \quad (4)$$

where L_{12}^{max} is the maximum power allowed to be transferred through the line between nodes 1 and 2. In addition, the demand profile is forecasted in advance but there will be some perturbation around the predicted profile. This can be described by a disturbance term in the load equation, and we assume a normal distribution for this term. We choose the demand model as follows:

$$D(t) = d(t) + w^d(t) \quad (5)$$

where D, d, and w^d represent the actual load profile, the load profile prediction, and demand prediction disturbance, respectively. Finally, the third node includes a micro gas turbine, whose amount of generation $P^{gas}(t)$ is commanded by MGCC. The generated power $P^{gas}(t)$ is transferred to the costumer node via the line between nodes 2 and 3. Hence, the constraint imposed by the line capacity is:

$$0 \leq P^{gas}(t) \leq L_{32}^{max} \quad (6)$$

where L_{32}^{max} is the maximum power allowed to be transferred through the line between the nodes 3 and 2. The graph shown in Figure 3 is a directed graph with two edges from nodes 1 and 3 to the node 2. The direction of edges shows the direction of power flowing from supply to demand in the MG. Obviously, the ultimate goal is for the MGCC to set the generation source power such that the supply could meet the demand. The latter statement can be mathematically described by:

$$P_1(t) + P^{gas}(t) = D(t) \quad (7)$$

This should be achieved so that the cost of generation be minimized and the battery SOC is kept as high as possible by scheduling the micro gas turbine generated power. To take these design objectives into account, we define the following cost function:

$$\begin{aligned} J &= \frac{1}{2}(SOC^{max} - SOC(T))^T P(SOC^{max} - SOC(T)) \\ &+ \sum_{t=0}^{T-1}[\frac{1}{2}(SOC^{max} - SOC(t))^T Q(SOC^{max} - SOC(t)) \\ &+ \frac{1}{2}(P^{gas}(t))^T RP^{gas}(t)] \end{aligned} \quad (8)$$

The above optimization problem is a finite horizon problem with T being the horizon length. As described before, we will formulate our grid management problem in an MPC form. First, we convert the above cost function to a standard MPC quadratic cost function form by changing the variable $SOC(t)$ as follows:

$$z(t) = SOC^{max} - SOC(t) \quad (9)$$

So, (2) is now rewritten as:

$$z(t+1) = z(t) + P^{batt}(t) \quad (10)$$

and (3) is written as:

$$0 \leq z(t) \leq SOC^{max}. \quad (11)$$

Consequently, the cost function in (8) becomes

$$\begin{aligned} J &= \frac{1}{2}z(T)^T Pz(T) \\ &+ \sum_{t=0}^{T-1}[\frac{1}{2}z(t)^T Qz(t) + \frac{1}{2}(P^{gas}(t))^T RP^{gas}(t)]. \end{aligned} \quad (12)$$

Therefore, for the microgrid management, the goal is to solve the following MPC problem:

$$\begin{aligned} \min_{P^{gas}} \quad & J \\ \text{subject to:} \quad & (1), (4-7), (10-11) \end{aligned} \quad (13)$$

In the next section, we will describe a solution method to the above stochastic model predictive control problem by taking advantage of dynamic programming.

III. Stochastic Model Predictive Control

In this section, we propose an algorithm to solve the stochastic model predictive control problem described in the previous section considering the input and state constraints. To this purpose, we first combine the constraints in (13) to reduce the number of constraints and put them in a compact form. Using (1), (5), (7), (10), and (11), we have:

$$z(t+1) = z(t) + d(t) + w^{d}(t) - P^{wind}(t) - w^{wind}(t) - P^{gas}(t). \tag{14}$$

In addition, by combining (1), (4), (6), and (7), we determine that:

$$max\{D(t) - L_{12}^{max}, 0\} \leq P^{gas}(t) \leq L_{32}^{max} \tag{15}$$

Therefore, the optimization problem to be solved is now in the following form:

$$\begin{aligned} \min_{P^{gas}} \quad & J \\ \text{subject to:} \quad & (11), (14), (15) \end{aligned} \tag{16}$$

The presence of the stochastic disturbance inputs in the problem under study brings a challenge in solving the MPC problem. Second challenge is due to the existence of input and state constraints resulting in the minimization of the cost function expected value over a finite horizon to become difficult to solve. In these situations, where an analytical solution does not exist, Monte Carlo methods have received attention in controls theory [4], [13], [17]. In the present paper, we propose an algorithm to solve the described MPC problem using *empirical mean* and *dynamic programming* tools to handle the constraints and expectation value computation.

A. Empirical Mean

We can appropriately approximate the expectation of a cost function, i.e., $\mathbb{E}J$, using its so-called empirical mean $\hat{\mathbb{E}}J$. By choosing n independent, identically distributed samples from w, empirical mean can be computed as follows:

$$\hat{\mathbb{E}}J = \frac{1}{n}\sum_{i=1}^{n} J(w_i). \tag{17}$$

Such an approximation is valid only if the error defined by $|\mathbb{E}J - \hat{\mathbb{E}}J|$ can be somehow evaluated. Due to the stochastic nature of the empirical mean function, this error can be measured in a probabilistic way. For example, we can guarantee with a probability of α that the estimation (17) has the accuracy γ if $|\mathbb{E}J - \hat{\mathbb{E}}J| < \gamma$. This lower bound on the probability, i.e., α, can be computed through Chebyshev inequality as follows [18]:

$$Prob(|\mathbb{E}J - \hat{\mathbb{E}}J| < \gamma) \geq 1 - \frac{\sigma(J)}{n\gamma^2} \tag{18}$$

where σ represents the variance. It is trivial to show that as $n \to \infty$, the empirical mean converges to the expected value.

B. Stochastic Model Predictive Control Algorithm

The well-known dynamic programming method can be employed to solve the optimization problem corresponding to the MPC problem backwards from $t = T$ to $t = 0$ at each horizon. For the stochastic systems, we have to compute the cost function expected value which is an easy task only for the first stage corresponding to $t = T$. However, this calculation is not straightforward for other stages since the expected value of the cost function is not a quadratic function anymore. Hence, by having a sufficient number of samples, empirical mean can be utilized as an appropriate alternative to estimate the expectation value. We describe below the steps of the proposed solution method for the stochastic MPC problem using the concept of empirical mean along with the dynamic programming method:

Step 1: Compute n using the Chebyshev inequality (18).

Step 2: To solve the DP problem using empirical mean, generate a sufficient number of disturbance samples and consequently state samples. To this end, we can arbitrarily extract n samples of disturbance at the first stage, i.e., $t = 0$. Having the initial conditions corresponding to state ($z(0)$) and control input ($P^{gas}(0) = 0$) along with the wind and load profiles, we can determine n possible $z(1)$, namely $z^i(1), i = 1, 2, ..., n$, through (14). For each $z^i(1)$, there will be n possible disturbance samples. So, we will have n^2 possible $z(2)$, namely $z^i(2), i = 1, 2, ..., n^2$. Continuing this process, we will have $n^T - 1$ possible $z(T-1)$ in the final stage. We call $z(0)$ the root node, $z^i(j)$ for $i = 1, ..., n^j, j = 1, ..., T-2$ intermediate nodes, and $z^i(T-1), i = 1, ..., n^{T-1}$ leaf nodes. Using the notation described, we have constructed a *tree* for stochastic dynamic programming stages based on disturbance samples.

Step 3: Solve the MPC problem starting from the last stage using the dynamic programming policy. To this purpose, compute the cost function for each leaf node as follows:

$$\begin{aligned} J(P^{gas}(T-1)) &= \frac{1}{2}z(T)^T P z(T) \\ &+ \frac{1}{2}z(T-1)^T Q z(T-1) \\ &+ \frac{1}{2}(P^{gas}(T-1))^T R P^{gas}(T-1) \end{aligned}$$

where $z(T)$ can be found in terms of $P^{gas}(T-1)$ using (14).

Step 4: Calculate the empirical mean over cost functions corresponding to each n leaf nodes connected to the same intermediate node.

Step 5: Minimize the empirical mean value of cost functions obtained in Step 4 over P^{gas}. The achieved optimal value will be the terminal cost for the next step.

Step 6: For each intermediate nodes $t = T - 2, ..., 1$, compute the cost function value using the terminal cost of Step 5, find the empirical mean over the obtained cost functions corresponding to the same node in their previous stage, minimize the achieved empirical value, and consider this minimum value as the terminal condition for previous stage.

Step 7: At $t = 0$, consider the calculated minimum value of the cost function as the minimum value of (12) at this iteration. If the difference between this value and the minimum value obtained in the previous iteration is lower than γ, the stochastic MPC problem has been solved and we choose this value as the optimal solution. Otherwise, based on the new value of P^{gas}, we update $z(t)$ for each node of tree and go back to Step 3.

IV. Simulation Results

In this section, we demonstrate the viability of the proposed microgrid management method described in the previous section using an illustrative example, where we first generate artificial data for MG graph and the corresponding cost function. It should be noted all the values reported in this section are converted to per unit (p.u.). For simulation purposes, we generate the daily power generation profile of the wind turbine in node 1 using a random distribution. This profile is shown in Figure 4(a). The uncertainty in the wind power forecast is also modeled by:

$$w^{wind}(t) \sim \mathcal{N}(0, 0.05).$$

In addition, maximum capacity of battery SOC^{max} is assumed to be 0.5, and maximum capacity of the distribution lines, i.e., L_{12}^{max} and L_{32}^{max}, is considered to be 1.3 p.u.

In node 2, we assume a number of costumers, whose daily load profile is also generated randomly for the simulation purposes. This profile is shown in Figure 4(b). In addition, the uncertainty in load prediction is described by:

$$w^{d}(t) \sim \mathcal{N}(0, 0.03).$$

The penalty coefficients in cost function are assumed to be:

$$P = 12, Q = 10, R = 12.$$

The first step in the algorithm described in the previous section is to find an acceptable number of samples n through Chebyshev inequality. Choosing $\gamma = 0.1$, we obtain $n = 7$. By having a sufficient number of samples, we can implement the stochastic MPC algorithm described before.

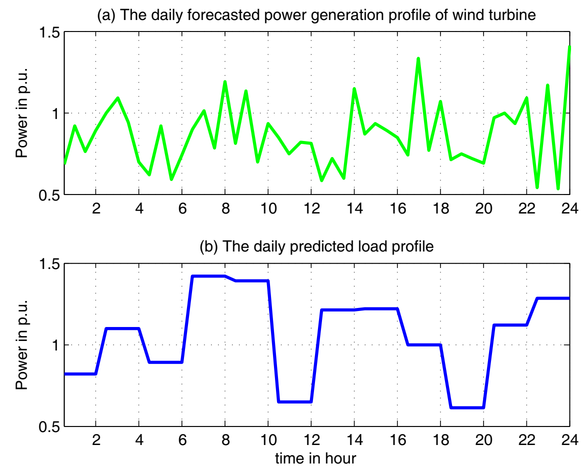

Fig. 4. (a) The daily forecasted power generation profile of wind turbine; (b) the daily predicted load profile

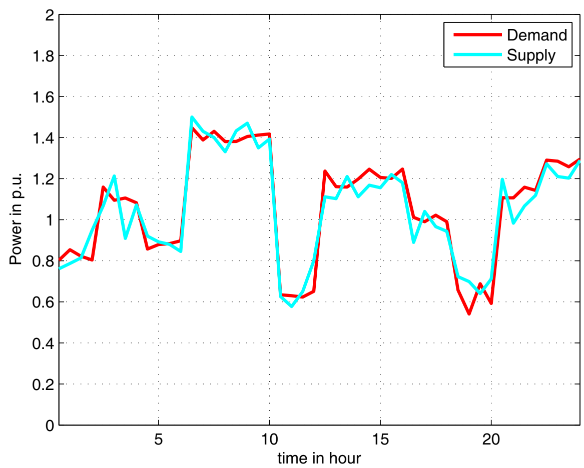

Fig. 5. Noisy load profile and optimized microgrid's total generated power (from wind turbine, battery and micro gas turbine)

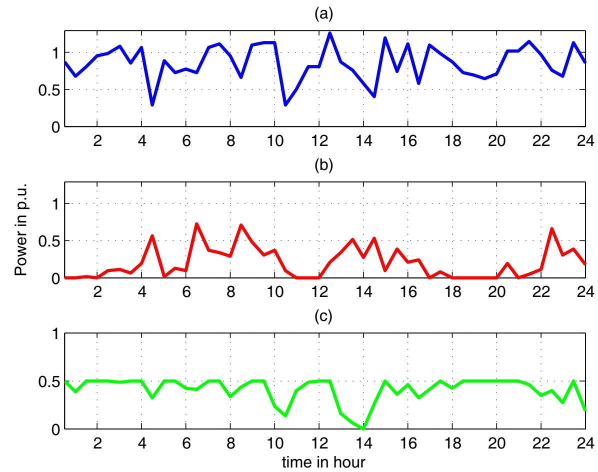

Fig. 6. (a) power generated by wind turbine and battery; (b) power generated by micro gas turbine; (c) battery state of charge

Due to the computational limitations in MATLAB, we set $T = 4$. The final calculated state, i.e., $z(4)$, will be considered as the initial state for the next set of data. In addition, we use IBM ILOG CPLEX Optimization Studio 12.2 [15] for performing minimizations required in steps 5 to 7. Since the defined cost function is convex, CPLEX determines an optimal solution. A *mex* function is coded and employed to link CPLEX and MATLAB. Next, we describe the performed analysis and simulation results.

Figure 5 illustrates the demand profile of costumer(s) in node 2, as well as the total power generated in MG by wind turbine and battery in node 1 and micro gas turbine in node 3. As observed, the generated power, i.e., supply, tracks the demand curve acceptably, and hence, the primary design objective is satisfied.

Shown in Figure 6(a) is the total power generated in node 1 of the microgrid that is supplied by the wind turbine and battery. This power is transferred to the costumer side through the line between nodes 1 and 2. As observed, the power is always below the maximum line capacity, i.e., $L_{12}^{max} = 1.3\ p.u.$ This implies that the line capacity constraint has also been met. We have shown in Figure 6(b) the power generated in node 3 supplied by the micro gas turbine. Examining this figure implies that the MPC algorithm has tried to keep this generation as low as possible as it was one of the objectives in the defined cost function. Furthermore, the power generated in this node is transferred to the costumer side through the line between nodes 3 and 2. As expected, the transferred power is always below the maximum line capacity, i.e., $L_{32}^{max} = 1.3\ p.u.$ Hence, the line capacity constraint between the nodes 2 and 3 is met as well. Finally, shown in Figure 6(c) is the battery state of charge SOC. As indicated before, one of the design objectives was to keep the SOC as close as possible to its maximum value (which is $0.5\ p.u.$ for the example discussed here). As illustrated in Figure 6(c), this goal is met as well.

V. Concluding Remarks

In this paper, we studied the problem of microgrid management considering a three-node topology for the power network. To this purpose, we defined a cost function along with the constraints to take into account the design specifications and formulated the design problem as an optimization problem. We then proposed a method based on the combination of empirical mean and dynamic programming to solve the stochastic model predictive control problem. An illustrative example was finally used to demonstrate the viability of the proposed microgrid management strategy.

The authors are currently examining the extension of this work to take into account the variation of gas and electricity price modeled as varying penalty coefficients in the design cost function. In addition, the realistic profiles of multiple customers load and generated power using wind turbines from the National Renewable Energy Lab (NREL) website are being employed to validate the proposed solution method. Further work includes the investigation of new methods to reduce the computational complexity involved in solving the stochastic MPC problem.

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