Automatica 81 (2017) 342–349

Contents lists available at ScienceDirect

Automatica

journal homepage: www.elsevier.com/locate/automatica

Brief paper

# A team-based approach for coverage control of moving sensor networks[☆]

Farshid Abbasi, Afshin Mesbahi, Javad Mohammadpour Velni

*School of Electrical & Computer Engineering, College of Engineering, The University of Georgia, Athens, GA 30602, USA*

ARTICLE INFO

*Article history:*
Received 1 April 2016
Received in revised form
8 January 2017
Accepted 22 March 2017

*Keywords:*
Coverage control
Team-based partitioning
Heterogeneous agents
Dynamic boundaries

ABSTRACT

The present paper proposes a new team-based approach that allows for forming multiple teams of agents within the coverage control framework. The objective function defined for this purpose tends to minimize the accumulative distance from each agent while reckoning with the given density function that defines the probability of events in the environment to be covered. The proposed team-based approach via the defined optimization problem allows for forming teams of agents when for a variety of reasons, e.g., heterogeneity in their embedded communication capabilities or the dynamics, it is required to keep the similar agents together in the same team. To realize this, the overall objective function is defined as the accumulated sensing cost of individual agents belonging to different teams. The defined collective cost function captures the interdependency of the team's Voronoi cells on the position of the agents that can be viewed as the impact of the dynamic boundaries on the agents. A gradient descent-based controller is designed to ensure the locally optimum configuration of the teams and agents within each team. The convergence of the proposed method is studied to ensure the stability of the implemented controller in both teams and agents final configuration. In addition, a new formation control approach is proposed using the team-based framework to impose either the same or different formation structures while performing the underlying coverage task. It is shown that maintaining the desired configuration through the proposed formation control is achieved at the cost of sacrificing the sensing performance. Finally, the proposed coverage and formation methods are examined via a numerical example where multiple heterogeneous teams of agents with potentially different number of agents are deployed.


## 1. Introduction

There have been advancements in developing techniques for deploying a group of robots in a given environment to perform assigned distributed tasks within the coverage framework. Examples of such tasks include surveillance, search and rescue operations, sensing, and data collection (Atınç, Stipanović, & Voulgaris, 2014; Lee, Diaz, & Egerstedt, 2015; Nowzari & Cortés, 2012). The previous works neglect the fact that the partitioning might be subjected to other constraints like the deployment of robots with various communication range or different embedded sensory devices. Furthermore, it might be required to assign different tasks with respect to the robots capability or even deploy robots in groups of agents each of which should carry out a specific task while collaborating with other agents. The existing approaches for the coverage control are based on the assumption that all agents belong to a single team (Patel, Frasca, & Bullo, 2013). The traditional Voronoi cells and their underlying objective function divide the main region among the agents as there are multiple individual agents with no consideration of their potential differences. However, this assumption is not realistic in many real-world applications, as the agents may differ from, e.g., dynamics or communication perspective (Sharifi, Chamseddine, Mahboubi, Zhang, & Aghdam, 2015; Stergiopoulos & Tzes, 2013). A multi-robot system can generally be considered as a homogeneous or heterogeneous system depending on the similarities or differences in their properties, e.g., desired performance index, dynamics, etc., that is required when coping with various complex assigned tasks as in Kantaros, Thanou, and Tzes (2015) and Song, Liu, Feng, and Xu (2016). As an example, it might be necessary to deploy agents equipped with different sensors to collect various types of data from the environment. In addition, the heterogeneity in dynamics may affect the relative distance of the

[☆] The material in this paper was partially presented at the 2016 American Control Conference, July 6–8, 2016, Boston, MA, USA. This paper was recommended for publication in revised form by Associate Editor Wei Ren under the direction of Editor Christos G. Cassandras.

*E-mail addresses:* fabbasi@uga.edu (F. Abbasi), mesbahi@uga.edu (A. Mesbahi), javadm@uga.edu (J. Mohammadpour Velni).

http://dx.doi.org/10.1016/j.automatica.2017.04.019

agents leading to a communication loss or sensing performance degradation. Hence, new flexible frameworks are needed to ensure that different types of heterogeneous agents can be utilized and operated in a distributed manner in an unknown, unstructured environment.

In the present work, a new coverage strategy is proposed that aims at taking into account the differences in the robots dynamics by offering a team-based design approach, where, each robot might team up with others based on its assigned task, associated dynamics or embedded communication capabilities. A different paradigm to tackle this problem is to define a two-level optimization problem at the teams and agents level independently (Abbasi, Mesbahi, & Velni, 2016). However, this framework neglects the dependency of the team boundaries on the position of the agents. Also, the proposed optimization problem does not account for imposing certain local formations. In the present paper, the underlying optimization problem is defined in such a way that it can allow for partitioning the teams and the agents collectively. This translates to taking into account the dependency of the teams' Voronoi cells on the position of the agents. This would make it possible to improve reliability and flexibility of the deployment algorithm. Throughout this work, it is assumed that the structure of the teams and the agents within each team is known a priori. The proposed approach addresses the problem of agents deployment by considering teams of robots instead of evaluating each agent individually. The agents in each team can be classified into two groups of interior members and members on the boundaries based on whether or not they share a boundary with the agents belonging to other teams.

As an application of the proposed team-based method, we study formation control problem via introducing a formation term into the performance function. The additional term ensures a certain distance from the nucleus by changing the formation factor. This factor enforces the agent to either expand or compress with respect to the desired formation. Different formations can be also achieved through selection of various formation factors.

**The contributions of the present work are threefold.** First, it is shown that incorporating the team concept into the coverage related tasks facilitates the deployment of multiple heterogeneous agents to handle multiple assigned tasks. Second, the agents can be allocated over a region in various teams with (possibly) different number of members each to address different scenarios like the case where a higher number of agents are needed to accomplish a certain assigned task. Finally, the proposed framework enables to impose local formations by giving a number of agents a team entity. This stems from the inevitable existence of constraints or certain objectives in practice, e.g., the limited communication radius or the need for a better coverage by maintaining a certain formation while ensuring an optimal coverage through collaborating with other teams.

The remainder of this paper is structured as follows. Definitions and the problem statement are provided for the team-based coverage control in Section 2. Section 3 introduces an approach for formation control. Section 4 presents numerical simulation results to illustrate the team-based partitioning and formation control.

### 1.1. Notations

We use $\mathbb{N}$, $\mathbb{R}$, and $\mathbb{R}_+$ to respectively denote the sets of natural, real, and nonnegative real numbers. Throughout the paper, $I_r$ denotes $r \times r$ identity matrix. We define $Q$ as a convex polytope in $\mathbb{R}^2$ and let $\mathcal{Q} = \{Q_1, Q_2, \ldots, Q_t\}$ be a *partition* of $Q$ as a collection of $t$ closed subsets with disjoint interiors. The boundary of $Q$ is denoted by $\partial Q$. Moreover, the so-called *distribution density function* is denoted by $\varphi$ where $\varphi : Q \to \mathbb{R}_+$ represents the probability of some phenomenon occurring over space $Q$. The function $\varphi$ is assumed to be measurable and absolutely continuous. The Euclidean distance function is denoted by $\| \cdot \|$, and $|Q|$ represents the Lebesgue measure of convex subset $Q$. The vector set $\mathcal{P}_t = (p_{t1}, p_{t2}, \ldots, p_{tn_t})$ is the location of $n_t$ agents belonging to $t$th team.

## 2. A team-based approach for coverage control

In this work, we present a modified version of the locational function that is suitable for the proposed team-based method. The team-based partitioning of the agents introduced in this paper addresses this by dividing agents into multiple teams pursuing assigned tasks.

### 2.1. Voronoi partitions

The main objective of this work is to adopt a team-based concept in the agents deployment and partitioning framework. To achieve this, we first need to define an optimization problem that can handle not only the deployment and partitioning tasks inside teams but also the partitioning inside the defined polytope $Q$. This optimization problem should consider the agents individual cost function, as well as their accumulated cost within their teams. To start with, we define the set of teams by $\mathcal{L} = (l_1, l_2, \ldots, l_n)$ where $l_t, t = 1 : n$, represents the nucleus of the $t$th team that is a function of the agents position in the associated team, i.e., $l_t = g(p_{t1}, p_{t2}, \ldots, p_{tn_t})$. The dependency of $l_t$ on the position of the agents is discussed later. Next, we partition the polytope $Q$ into a set of Voronoi cells $\mathcal{V}(\mathcal{L}) = \{V_1, V_2, \ldots, V_n\}$ considered as the optimal partitioning for a set of agents with fixed locations at a given area as $V_t = \{q \in Q \mid \|q - l_t\| \le \|q - l_s\|, s = 1, \ldots, n; s \ne t\}$. The obtained Voronoi cells associated with the nuclei of the teams are then considered as the convex polytope set to deploy their associated agents. Therefore, the sub-partitions are defined on the basis of the Voronoi cells $V_t$ obtained from the team level partitioning. The Voronoi partitions $\mathcal{V}_t(\mathcal{P}_t) = \{V_{t1}, V_{t2}, \ldots, V_{tn_t}\}$ generated by the agents $(p_{t1}, p_{t2}, \ldots, p_{tn_t})$ belonging to the $t$th team are defined as

$$V_{tm} = \{q \in V_t \mid \|q - p_{tm}\| \le \|q - p_{tr}\|, r = 1, \ldots, n_t, r \ne m\}, \quad (1)$$

where $p_{tm}$ denotes the location of $m$th agent in $t$th team for $m \in \{1, \ldots, n_t\}$. The agents in each team are divided into two subgroups, boundary and interior groups, where the cells associated with each group require a different set of data, i.e., their neighbors' position, to be maintained. The interior group represents the agents that share boundaries only with the agents belonging to the same team while the agents in the boundary group have neighbors not only in the same team but also in the neighboring teams—they may also share boundaries with the convex polytope $Q$. In general, a boundary associated with each agent $\partial V_{tm}$ is either an edge shared with the agents within the same team or edges shared with the teams in the neighborhood depending on the position of the agent within the team. The agents in the boundary group share at least one edge with other teams. An edge that is shared with the neighboring agent $f$ in the same team is shown by $\partial V_{tm,f}$. The edges associated with the agents in the boundary group shared with the neighboring team $k$ and the main convex polytope $Q$ are represented by $\partial V_{tb}^k$ and $\partial V_{tb}^0$, respectively. Fig. 1 illustrates the boundaries and their normal vectors for Voronoi $V_{tm}$. It is noted that the agents in the boundary group may share boundaries with the agents in the interior group where the same notation as the boundaries of the interior agents is used to represent these edges. We recall the basic characteristics of the Voronoi partitions including their associated mass and centroid defined as $M_{V_{tm}} = \int_{V_{tm}} \varphi(q) \mathrm{d}q$ and $C_{V_{tm}} =$

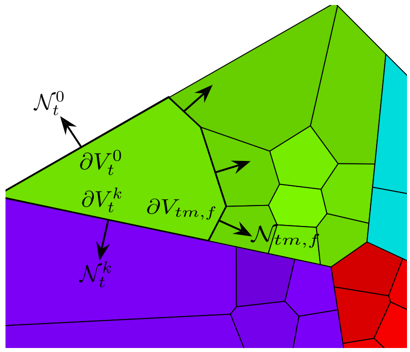


**Fig. 1.** The boundaries and their associated normal vectors for the Voronoi cell $V_{tm}$ and the edges shared with both team $k$ in its neighborhood and the agent located at $p_{tf}$, i.e., $f$th agent in the $t$th team, in the same team in addition to the edge $\partial V_t^0$ shared with the main region $Q$.

$\frac{1}{M_{V_{tm}}} \int_{V_{tm}} q \ \ \varphi(q)\mathrm{d}q$ (Cortes, Martinez, Karatas, & Bullo, 2004). In addition to the parameters defined for the Voronoi cells of each agent inside the teams, we also need to define the characteristics of the teams' Voronoi cells. According to the definition of the Voronoi partitions, it can be deduced that $M_{V_t} = \sum_{m=1}^{n_t} M_{V_{tm}}$ and $C_{V_t} = \frac{1}{M_{V_t}} \int_{V_t} q \ \varphi(q)\mathrm{d}q$. The nucleus of the team is defined as

$$l_t = \frac{\sum_{m=1}^{n_t} M_{V_{tm}} p_{tm}}{\sum_{m=1}^{n_t} M_{V_{tm}}}, \tag{2}$$

which is a representative of the agents position in the team and can be considered as the collective position of the agents. This measure is needed for the computation of the Voronoi diagram of the team $V_t$.

### 2.2. Formulation of the local optimization problem

The deployment task in the team-based framework can be addressed by solving an optimization problem over the convex polytope $Q$ where each team's region needs to be optimized in the sense of its associated sensing function. Hence, the following cost function is considered

$$\mathcal{G}(\mathcal{L}, \mathcal{Q}) = \sum_{t=1}^{n} \mathcal{G}_t(\mathcal{P}_t, \mathcal{Q}_t), \tag{3}$$

where $\mathcal{G}_t$ is the cost function associated with the sensing performance of the agents belonging to the $t$th team that is considered as $f(\|q - p_{tm}\|) = \|q - p_{tm}\|^2$ for the $m$th agent belonging to the $t$th team. The solution to (3) gives a local minimum to the deployment problem where agents are considered as the members of various collaborating teams. We define a set of polygons as $\mathcal{Q}_t = \{Q_{t1}, Q_{t2}, \ldots, Q_{tn_t}\}$, with disjoint interiors, whose union is $V_t$. The following sensing cost is defined for each team as a function of the position of its associated agents

$$\mathcal{G}_t(\mathcal{P}_t, \mathcal{Q}_t) = \sum_{m=1}^{n_t} \int_{Q_{tm}} \|q - p_{tm}\|^2 \varphi(q)\mathrm{d}q. \tag{4}$$

**Remark 1.** It is proven that among different partitioning schemes, the Voronoi partitions are optimum (for a single team) in the sense of minimizing the defined cost function (4) (Cortes et al., 2004). Hence, for a given set of agents with position $\mathcal{P}_t \in V_t$ and a *partition* $\mathcal{Q}_t$ of $V_t$, we have $\mathcal{G}_t(\mathcal{P}_t, \mathcal{V}_t(\mathcal{P}_t)) \leq \mathcal{G}_t(\mathcal{P}_t, \mathcal{Q}_t)$, which implies that the Voronoi cells represent the optimum partitioning of the area associated with each team.

The next step is to obtain (locally optimum) location of the agents and the nucleus of their associated team. The derivative with respect to the coordinates of agent in $p_{sm}$ is obtained as

$$\begin{aligned} \frac{\partial \mathcal{G}}{\partial p_{sm}} &= \int_{V_{sm}} \frac{\partial}{\partial p_{sm}} \|q - p_{sm}\|^2 \varphi(q)\mathrm{d}q \\ &\quad + \sum_{t=1}^{n} \Bigg( \sum_{i=1}^{n_{it}} \int_{\partial V_{ti}} \|q - p_{ti}\|^2 \varphi(q) \frac{\partial \partial V_{ti}}{\partial p_{sm}} \mathcal{N}_{ti} \mathrm{d}q \\ &\quad + \sum_{b=1}^{n_{bt}} \int_{\partial V_{tb}} \|q - p_{tb}\|^2 \varphi(q) \frac{\partial \partial V_{tb}}{\partial p_{sm}} \mathcal{N}_{tb} \mathrm{d}q \Bigg), \end{aligned} \tag{5}$$

where $n_{it}$ is the number of the interior agents, $n_{bt}$ is the number of agents in the boundary group in the $t$th team and $s = 1, \ldots, n, \ m = 1, \ldots, n_t$. It can be inferred that the boundaries of the Voronoi cell $V_{tr}$ are either directly or indirectly dependent on $p_{tm}$. For the sake of clarity, the location of the interior and boundary agents in team $t$ are shown by $p_{ti}$ and $p_{tb}$, where $i = 1, \ldots, n_{it}$ and $b = 1, \ldots, n_{bt}$, respectively. The direct dependency is obviously resulted from the definition of the interior Voronoi cells (1). For the agents in the interior group of agents $p_{ti}$, we note that $V_{ti}$ and the Voronoi cells in its neighborhood are directly dependent on $p_{ti}$. The indirect dependency can be seen in the boundary agents $p_{tb}$, where they share at least one edge with the agents in the neighboring teams (or contribute at least one edge to the boundary of the team).

**Remark 2.** The boundaries shared with other teams are indirectly dependent on $p_{tm}$ via the definition of the nucleus of the team $l_t$. Also, the edges that are shared with other Voronoi cells $V_{tf}$ in the same team $t$ are independent of the position of the agents in other teams $p_{sm}$ leading to $\frac{\partial \partial V_{tr,f}}{\partial p_{sm}} = 0$ for $t \neq s$.

Two possible scenarios are studied here. First, the agent $p_{sm}$ is assumed to belong to the interior group of agents. Second, it is considered to be in the boundary group. From Remark 2, Eq. (5) can be reduced to taking the integral on the boundaries of $V_{sm}$ and the ones that it shares boundaries with for both cases as follows

$$\begin{aligned} &\sum_{i=1}^{n_{is}} \int_{\partial V_{si}} \|q - p_{si}\|^2 \varphi(q) \frac{\partial \partial V_{si}}{\partial p_{sm}} \mathcal{N}_{si} \mathrm{d}q \\ &\quad + \sum_{b=1}^{n_{bs}} \int_{\partial V_{sb}} \|q - p_{sb}\|^2 \varphi(q) \frac{\partial \partial V_{sb}}{\partial p_{sm}} \mathcal{N}_{sb} \mathrm{d}q \\ &= \sum_{f \in \mathcal{F}} \int_{\partial V_{sm,f}} \|q - p_{sm}\|^2 \varphi(q) \frac{\partial \partial V_{sm,f}}{\partial p_{sm}} \mathcal{N}_{sm,f} \mathrm{d}q \\ &\quad + \sum_{f \in \mathcal{F}} \int_{\partial V_{sf,m}} \|q - p_{sf}\|^2 \varphi(q) \frac{\partial \partial V_{sf,m}}{\partial p_{sm}} \mathcal{N}_{sf,m} \mathrm{d}q \\ &\quad + \sum_{t \in \{s, g | l_g \in \mathcal{N}_{l_s}\}} \sum_{b=1, k \in \mathcal{K}}^{n_{bt}} \int_{\partial V_{tb}^k} \|q - p_{tb}\|^2 \varphi(q) \frac{\partial \partial V_{tb}^k}{\partial p_{sm}} \mathcal{N}_{tb}^k \mathrm{d}q, \end{aligned} \tag{6}$$

where $\mathcal{N}_{sf,m}$ is the normal vector associated with the edge $\partial V_{sf,m}$, and $\mathcal{F} = \{f | p_{sf} \in \mathcal{N}_{p_{sm}}\}$ with $\mathcal{N}_{p_{sm}}$ representing the set of agents that share boundaries with agent $p_{sm}$. Also, $\mathcal{K} = \{0, g | l_g \in \mathcal{N}_{l_t}\}$ and $\mathcal{N}_{l_t}$ is the set of teams neighboring with team $t$. Moreover, $\mathcal{N}_{tb}^k$ in (6) denotes the normal vector associated with the edge $\partial V_{tb}^k$. It should be noted that the agent $p_{sf}$ may belong to either the interior or the boundary groups.

**Remark 3.** The integral on each boundary shared with the neighboring agents is identical for agents on both sides except that the normals will have opposite signs, i.e., $\mathcal{N}_{sm,f} = -\mathcal{N}_{sf,m}$.

$$\frac{\partial l_s}{\partial p_{sm}}\Big(\Big(\int_{V_s}\varphi(q)\mathrm{d}q\Big)I_2+\Big(\sum_{k\in\mathcal{K}'}\int_{\partial V_s^k}\varphi(q)\frac{\partial\partial V_s^k}{\partial l_s}\mathcal{N}_s^k\mathrm{d}q\Big)l_s^\top-\sum_{b=1,k\in\mathcal{K}'}^{n_{bs}}\Big(\int_{\partial V_{sb}^k}\varphi(q)\frac{\partial\partial V_{sb}^k}{\partial l_s}\mathcal{N}_{sb}^k\mathrm{d}q\Big)p_{sb}^\top\Big)$$
$$=\Big(\int_{V_{sm}}\varphi(q)\mathrm{d}q\Big)I_2+\sum_{p_{sr}\in\{p_{sm},\mathcal{N}_{p_{sm}}\}}\Big(\int_{\partial V_{sr}}\varphi(q)\frac{\partial\partial V_{sr}}{\partial p_{sm}}\mathcal{N}_{sr}\mathrm{d}q\Big)p_{sr}^\top, \tag{7}$$

$$\frac{\partial l_s}{\partial p_{sm}}\Big(\Big(\int_{V_s}\varphi(q)\mathrm{d}q\Big)I_2+\Big(\sum_{k\in\mathcal{K}'}\int_{\partial V_s^k}\varphi(q)\frac{\partial\partial V_s^k}{\partial l_s}\mathcal{N}_s^k\mathrm{d}q\Big)l_s^\top\Big)=\Big(\int_{V_{sm}}\varphi(q)\mathrm{d}q\Big)I_2+\sum_{r=1}^{n_s}\Big(\int_{\partial V_{sr}}\varphi(q)\frac{\partial\partial V_{sr}}{\partial p_{sm}}\mathcal{N}_{sr}\mathrm{d}q\Big)p_{sr}^\top. \tag{8}$$

**Box I.**

Furthermore, we have $\|q-p_{sm}\|=\|q-p_{sf}\|$ for the points on the shared boundaries. Hence

$$\sum_{f\in\mathcal{F}}\int_{\partial V_{sm,f}}\|q-p_{sm}\|^2\varphi(q)\frac{\partial\partial V_{sm,f}}{\partial p_{sm}}\mathcal{N}_{sm,f}\mathrm{d}q$$
$$=-\sum_{f\in\mathcal{F}}\int_{\partial V_{sf,m}}\|q-p_{sm}\|^2\varphi(q)\frac{\partial\partial V_{sf,m}}{\partial p_{sm}}\mathcal{N}_{sf,m}\mathrm{d}q. \tag{9}$$

The first two terms on the right hand side of (6) represent the integral on the shared boundaries of the $V_{sm}$ with the agents belonging to the same team. Hence, it can be concluded from Remark 3 that these terms cancel each other out. Moreover, the boundaries shared with $\partial Q$ have no dynamics that results in $\frac{\partial\partial V_{tb}^0}{\partial p_{sm}}=0$. Due to the dependency of the team boundaries on the nucleus, only the boundaries of the agents in the boundary group shared with their associated team should be considered for the integration. Substituting (9) into (6) results in

$$\frac{\partial\mathcal{G}}{\partial p_{sm}}=\int_{V_{sm}}\frac{\partial}{\partial p_{sm}}\|q-p_{sm}\|^2\varphi(q)\mathrm{d}q$$
$$+\sum_{t\in\{s,g|l_g\in\mathcal{N}_{l_s}\}}\sum_{b=1,k\in\mathcal{K}'}^{n_{bt}}\int_{\partial V_{tb}^k}\|q-p_{tb}\|^2\varphi(q)\frac{\partial\partial V_{tb}^k}{\partial p_{sm}}\mathcal{N}_{tb}^k\mathrm{d}q, \tag{10}$$

where $\mathcal{K}'=\{s|l_s\in\mathcal{N}_{l_t}\}$. Next, to calculate the derivative terms appearing because of the dependency of the exterior boundaries on the nucleus, the chain rule can be applied as follows

$$\frac{\partial\partial V_{tb}^k}{\partial p_{sm}}=\frac{\partial l_s}{\partial p_{sm}}\frac{\partial\partial V_{tb}^k}{\partial l_s}, \tag{11}$$

where using (2) and (11), we obtain

$$\frac{\partial}{\partial p_{sm}}\Big(l_s\int_{V_s}\varphi(q)\mathrm{d}q\Big)=\frac{\partial}{\partial p_{sm}}\Big(\sum_{r=1}^{n_s}p_{sr}\int_{V_{sr}}\varphi(q)\mathrm{d}q\Big). \tag{12}$$

By taking the derivatives from both sides and by applying the chain rule (11), we have

$$\frac{\partial l_s}{\partial p_{sm}}\Big(\int_{V_s}\varphi(q)\mathrm{d}q\Big)I_2+\frac{\partial}{\partial p_{sm}}\Big(\int_{V_s}\varphi(q)\mathrm{d}q\Big)l_s^\top$$
$$=\Big(\int_{V_{tm}}\varphi(q)\mathrm{d}q\Big)I_2+\sum_{r=1}^{n_s}\frac{\partial}{\partial p_{sm}}\Big(\int_{V_{sr}}\varphi(q)\mathrm{d}q\Big)p_{sr}^\top. \tag{13}$$

Also, the last term of (13) can be written as follows

$$\sum_{r=1}^{n_s}\frac{\partial}{\partial p_{sm}}\Big(\int_{V_{sr}}\varphi(q)\mathrm{d}q\Big)p_{sr}^\top$$
$$=\sum_{r=1}^{n_s}\Big(\int_{\partial V_{sr}}\varphi(q)\frac{\partial\partial V_{sr}}{\partial p_{sm}}\mathcal{N}_{sr}\mathrm{d}q\Big)p_{sr}^\top. \tag{14}$$

By substituting (14) into (13), we obtain (8) (given in Box I). The last term in (8) represents the variation of the boundaries of the Voronoi cells with respect to $p_{sm}$. As described earlier, the edges of the Voronoi cells $V_{sm}$ and the Voronoi cells of the agents in its neighborhood shared with $p_{sm}$ are directly dependent on $p_{sm}$. In addition to this, the edges of the boundary Voronoi cells shared with the team boundaries are indirectly dependent on $p_{sm}$. This indirect dependency is due to the computation of the team boundaries through the nuclei of the teams and their dependency on $p_{sm}$ through (2). Hence, using the chain rule, we have

$$\sum_{r=1}^{n_s}\Big(\int_{\partial V_{sr}}\varphi(q)\frac{\partial\partial V_{sr}}{\partial p_{sm}}\mathcal{N}_{sr}\mathrm{d}q\Big)p_{sr}^\top$$
$$=\sum_{p_{sr}\in\{p_{sm},\mathcal{N}_{p_{sm}}\}}\Big(\int_{\partial V_{sr}}\varphi(q)\frac{\partial\partial V_{sr}}{\partial p_{sm}}\mathcal{N}_{sr}\mathrm{d}q\Big)p_{sr}^\top$$
$$+\sum_{b=1,k\in\mathcal{K}'}^{n_{bs}}\Big(\int_{\partial V_{sb}^k}\varphi(q)\frac{\partial l_s}{\partial p_{sm}}\frac{\partial\partial V_{sb}^k}{\partial l_s}\mathcal{N}_{sb}^k\mathrm{d}q\Big)p_{sb}^\top. \tag{17}$$

By substituting (17) into (8), we obtain (7) (given in Box I) where $n_{bs}$ is the number of the boundary agents in the $s$th team. The effect of the moving boundaries with respect to the variation of the nucleus is shown by the following

$$M_{\partial V_{sr}}^k=\int_{\partial V_{sr}^k}\varphi(q)\frac{\partial\partial V_{sr}^k}{\partial l_s}\mathcal{N}_{sr}^k\mathrm{d}q, \tag{18}$$

$$M_{\partial V_s}=\sum_{b=1,k\in\mathcal{K}'}^{n'_{bs}}M_{\partial V_{sb}}^k, \tag{19}$$

and the changing boundary of the interior agents due to the agents dynamics is represented by

$$M_{\partial V_{sr}}=\int_{\partial V_{sr}}\varphi(q)\frac{\partial\partial V_{sr}}{\partial p_{sm}}\mathcal{N}_{sr}\mathrm{d}q. \tag{20}$$

Using these notations results in the following

$$\frac{\partial l_s}{\partial p_{sm}}=\Big(M_{V_{sm}}I_2+\sum_{p_{sr}\in\{p_{sm},\mathcal{N}_{p_{sm}}\}}M_{\partial V_{sr}}p_{sr}^\top\Big)$$
$$\times\Big(M_{V_s}I_2+M_{\partial V_s}l_s^\top-\sum_{b=1,k\in\mathcal{K}'}^{n'_{bs}}M_{\partial V_{sb}}^k p_{sb}^\top\Big)^{-1}. \tag{21}$$

The points on the team boundaries are represented by

$$(\mathcal{N}_s^k)^\top\Big(q-\frac{l_k+l_s}{2}\Big)=0,\quad q\in\partial V_s^k, \tag{22}$$

where $\partial V_s^k$ is the boundary shared between teams $s$ and $k$, and its associated normal vector $\mathcal{N}_s^k$ is obtained by

$$\mathcal{N}_s^k=\frac{l_k-l_s}{\|l_k-l_s\|}. \tag{23}$$

$$\frac{\partial \mathcal{G}}{\partial p_{sm}} = \int_{V_{sm}} \frac{\partial}{\partial p_{sm}} \|q - p_{sm}\|^2 \varphi(q) \mathrm{d}q + \sum_{t \in \{s,g | l_g \in \mathcal{N}_{l_s}\}} \sum_{b=1,k\in\mathcal{K}'}^{n_{bt}} \int_{\partial V_{tb}^k} \|q - p_{tb}\|^2 \varphi(q) \frac{\partial l_s}{\partial p_{sm}} \frac{\partial \partial V_t^k}{\partial l_s} \mathcal{N}_t^k \mathrm{d}q. \tag{15}$$

$$\begin{aligned}\frac{\partial \mathcal{G}}{\partial p_{sm}} &= -2M_{V_{sm}}(C_{V_{sm}} - p_{sm}) + \Bigg(M_{V_{sm}} I_2 + \sum_{p_{tr} \in \{p_{sm}, \mathcal{N}_{p_{sm}}\}} M_{\partial V_{sr}} p_{sr}^\top\Bigg)\Bigg(M_{V_s} I_2 + M_{\partial V_s} l_s^\top - \sum_{b=1,k\in\mathcal{K}'}^{n_{bs}} M_{\partial V_{sb}}^k p_{sb}^\top\Bigg)^{-1} \\ &\quad \times \Bigg(\sum_{b=1,k\in\mathcal{K}'}^{n_{bs}} \Bigg(\frac{\mathcal{N}_s^k (\mathcal{N}_s^k)^\top - I_2}{\|l_s - l_k\|} \int_{\partial V_{sb}^k} \|q - p_{sb}\|^2 \varphi(q) \left(\frac{l_k + l_s}{2} - q\right) \mathrm{d}q + \frac{1}{2}\mathcal{N}_s^k \int_{\partial V_{sb}^k} \|q - p_{sb}\|^2 \varphi(q) \mathrm{d}q\Bigg) \\ &\quad + \sum_{t \in \{g | l_g \in \mathcal{N}_{l_s}\}} \sum_{b \in \mathcal{E}} \Bigg(\frac{I_2 - \mathcal{N}_t^s (\mathcal{N}_t^s)^\top}{\|l_t - l_s\|} \int_{\partial V_{tb}^s} \|q - p_{tb}\|^2 \varphi(q) \left(\frac{l_s + l_t}{2} - q\right) \mathrm{d}q + \frac{1}{2}\mathcal{N}_t^s \int_{\partial V_{tb}^s} \|q - p_{tb}\|^2 \varphi(q) \mathrm{d}q\Bigg)\Bigg). \end{aligned} \tag{16}$$

**Box II.**

The partial derivative of (22) with respect to $l_s$ is obtained as

$$\frac{\partial \mathcal{N}_s^k}{\partial l_s} \left(q - \frac{l_k + l_s}{2}\right) + \left(\frac{\partial \partial V_s^k}{\partial l_s} - \frac{1}{2}\right) \mathcal{N}_s^k = 0, \tag{24}$$

where

$$\frac{\partial \mathcal{N}_s^k}{\partial l_s} = \frac{\mathcal{N}_s^k (\mathcal{N}_s^k)^\top - I_2}{\|l_k - l_s\|}. \tag{25}$$

Substituting (25) into (24), we have

$$\frac{\partial \partial V_s^k}{\partial l_s} \mathcal{N}_s^k = \frac{\mathcal{N}_s^k (\mathcal{N}_s^k)^\top - I_2}{\|l_k - l_s\|} \left(\frac{l_k + l_s}{2} - q\right) + \frac{1}{2}\mathcal{N}_s^k, \tag{26}$$

where $q \in \partial V_s^k$. Because of the following equality that holds for the variation of the boundary edge $\partial V_{tb}^k$ with respect to the variation of the nucleus $l_s$

$$\frac{\partial \partial V_{tb}^k}{\partial l_s} \mathcal{N}_{tb}^k = \frac{\partial \partial V_t^k}{\partial l_s} \mathcal{N}_t^k, \tag{27}$$

the derivative (10) can be rewritten as (15) (given in Box II). Substituting (26) into (15) and the derivative of the first term implementing equations of mass and centroid results in (16) (given in Box II) where $\mathcal{E} = \{r | V_{tr} \cap \partial V_t^s \neq \emptyset\}$. As it can be seen, the boundaries of each team and its neighboring teams may vary with respect to the variation of the agent's location $p_{sm}$. This accounts for the sensitivity of the team boundaries with respect to any change in the nucleus of its associated team or the neighboring teams resulted from a change in agents' position. The term associated with the integration on the team boundaries in (16) is the same for all the agents in the $t$th team. In fact, at every time step, it will be calculated just once for all the agents $b \in \{1, \ldots, n_{bt}\}$ with exterior boundaries shared with the neighboring teams.

### 2.3. Computation of the Voronoi cells

The Voronoi cells associated with each agent and team require a set of information to be computed online. As described before, the agents in the interior group are able to compute the Voronoi cell by communicating with the neighboring agents within the same team. However, the agents on the boundary group need the position of the nucleus of their team, as well as the nuclei of their neighboring teams. A greedy algorithm that provides the required data flow within each team and between the so-called leaders of different teams has been proposed in our recent work (Abbasi et al., 2016). This is achieved at the cost of communicating with agents that are further away from each other; however, it allows the deployment of the agents without requiring the continuous communication among all the agents, which translates to less data exchange.

### 2.4. Controller design

A gradient descent-based control law is proposed here to guarantee the convergence of the teams of agents to their equilibrium point. The following dynamics is imposed on each agent

$$\begin{aligned}\dot{p}_{sm} &= u_{sm} = \frac{K_{sm}}{2M_{V_{sm}}}\left(-\frac{\partial \mathcal{G}}{\partial p_{sm}}\right) \\ &= \frac{K_{sm}}{2M_{V_{sm}}}\Big(2M_{V_{sm}}(C_{V_{sm}} - p_{sm}) - \gamma_{sm}\Big)\end{aligned} \tag{29}$$

where $s = 1, \ldots, n,\ \ m = 1, \ldots, n_s$, $K_{sm}$ is a positive scalar and $\gamma_{sm}$ contains all the remaining terms in (16). By dividing $K_{sm}$ by the varying term $2M_{V_{sm}}$, the control input is normalized to distribute the effect of both $2M_{V_{sm}}(C_{V_{sm}} - p_{sm})$ and $\gamma_{sm}$ in the controller design. While the first term drives the agent towards its centroid, the second term is associated with the changing boundaries of the teams of agents. In other words, the above control law ensures that the agents are confined within the given dynamic boundaries of their team.

### 2.5. Convergence of the proposed controller

The proposed controller drives the agents to their local optimum while taking into account the moving boundaries. To ensure the convergence of the agents to their collective local optimum, Lemma 2.1 is proposed.

**Lemma 2.1.** *The agents with the assigned dynamics (29) converge to a local minimum by imposing the control law proposed in (29). That is,*

$$\lim_{\tau \to \infty} \left\| -2M_{V_{sm}}(\tau)(C_{V_{sm}}(\tau) - p_{sm}(\tau)) + \gamma_{sm}(\tau) \right\| = 0, \tag{30}$$

*for* $\forall s \in \{1, \ldots, n\}, \forall m \in \{1, \ldots, n_t\}$.

**Proof.** The asymptotic behavior cannot be proved through invoking standard invariant set theorems for time-varying systems. We hence use the Barbalat's lemma to show the asymptotic convergence of the system when each agent follows an optimal configuration. To this aim, the Lyapunov-like function associated with each team is defined as

$$V = \mathcal{G}. \tag{31}$$

$$\lim_{\tau\to 0}\left\|\sum_{m=1}^{n_s}\left(M_{V_{sm}}(\tau)I_2+\sum_{p_{sr}\in\{p_{sm},\mathcal{N}_{p_{sm}}\}}M_{\partial V_{sr}}(\tau)p_{sr}^{\top}(\tau)\right)\left(M_{V_s}(\tau)I_2+M_{\partial V_s}(\tau)l_s^{\top}(\tau)-\sum_{b=1,k\in\mathcal{K}'}^{n_{bs}}M_{\partial V_{sb}}^{k}(\tau)p_{sb}^{\top}(\tau)\right)^{-1}\dot{p}_{sm}(\tau)\right\|=0. \quad (28)$$

**Box III.**

The derivative of this function is obtained as

$$\dot{V}=\sum_{s=1}^{n}\sum_{m=1}^{n_s}\left(\frac{\partial p_{sm}}{\partial\tau}\right)^{\top}\frac{\partial\mathcal{G}}{\partial p_{sm}}. \quad (32)$$

Substituting (16) and (29) into (32), we obtain

$$\dot{V}=-\sum_{s=1}^{n}\sum_{m=1}^{n_t}\frac{K_{sm}}{2M_{V_{sm}}}\left(-2M_{V_{sm}}(C_{V_{sm}}-p_{sm})+\gamma_{sm}\right)^{\top}\times\left(-2M_{V_{sm}}(C_{V_{sm}}-p_{sm})+\gamma_{sm}\right). \quad (33)$$

Since $M_{V_{sm}}$ and $K_{sm}$ are positive scalars, it can be concluded that the derivative (32) is non-positive, $\dot{V}\le 0$. Due to the positivity of the cost function $\mathcal{G}$, it is also concluded that the Lyapunov-like function (31) is non increasing and hence lower bounded. As shown in Schwager, Rus, and Slotine (2009), $\ddot{V}(\tau)$ is uniformly bounded that results in the uniform continuity of $\dot{V}(\tau)$. Next, since $V(\tau)$ is bounded and due to the continuity of $\dot{V}(\tau)$, it is proven by Barbalat's lemma that $\lim_{\tau\to\infty}\dot{V}=0$, hence

$$\lim_{\tau\to\infty}\|-2M_{V_{sm}}(\tau)(C_{V_{sm}}(\tau)-p_{sm}(\tau))+\gamma_{sm}(\tau)\|=0, \quad (34)$$

for $\forall s\in\{1,\dots,n\}$, $\forall m\in\{1,\dots,n_t\}$.

Due to the dependency of the nucleus of the $t$th team on the position of its agents, $l_t(p_{t1},\dots,p_{tm})$, Theorem 2.2 guarantees the convergence of the nuclei.

**Theorem 2.2.** *According to (2) and the dynamics of the nuclei $l_t$, it can be inferred from Lemma 2.1 that the nuclei will also converge to the local minimum of the team level optimization with the cost function defined in (3).*

**Proof.** The dynamics imposed on the nuclei of the teams are indirectly resulted from (29). This can be seen by taking the time derivative of (2) as

$$\frac{\partial l_s}{\partial\tau}=\sum_{m=1}^{n_s}\frac{\partial l_s}{\partial p_{sm}}\dot{p}_{sm}. \quad (35)$$

Substituting (21) into (35), we obtain

$$\frac{\partial l_s}{\partial\tau}=\sum_{m=1}^{n_s}\left(M_{V_{sm}}I_2+\sum_{p_{sr}\in\{p_{sm},\mathcal{N}_{p_{sm}}\}}M_{\partial V_{sr}}p_{sr}^{\top}\right)\times\left(M_{V_s}I_2+M_{\partial V_s}l_s^{\top}-\sum_{b=1,k\in\mathcal{K}'}^{n_{bs}}M_{\partial V_{sb}}^{k}p_{sb}^{\top}\right)^{-1}\dot{p}_{sm}. \quad (36)$$

It can be seen that when the agents inside a team converge to their associated local minima according to Lemma 2.1, the nuclei of the teams will also asymptotically converge to their local minima according to (28) (given in Box III).

## 3. Formation control of the teams of agents

The idea of team-based partitioning relies on forming teams of agents with respect to their capabilities and dynamics. It is highly likely that agents require to take different formations within their confined region while performing the assigned coverage task(s). A major factor in changing the formation of the agents is to change their relative distances. We introduce a formation term within each team while aiming at achieving the main partitioning task. The following cost function is defined to ensure that the agents can maintain a certain formation throughout their mission

$$\mathcal{G}_t(\mathcal{P}_t,\mathcal{Q}_t)=\sum_{m=1}^{n_t}\left(\int_{V_{tm}}\|q-p_{tm}\|^2\varphi(q)\mathrm{d}q+\alpha_{tm}\|p_{tm}-l_t\|^2\right), \quad (37)$$

in which the formation factor $\alpha_{tm}$ is a positive scalar. To solve the associated optimization problem, the derivative of (37) is obtained as

$$\frac{\partial\mathcal{G}}{\partial p_{sm}}=\int_{V_{sm}}\frac{\partial}{\partial p_{sm}}\|q-p_{sm}\|^2\varphi(q)\mathrm{d}q+\sum_{t\in\{s,g|l_g\in\mathcal{N}_{l_s}\}}\sum_{b=1,k\in\mathcal{K}'}^{n_{bt}}\int_{\partial V_{tb}^{k}}\|q-p_{tb}\|^2\varphi(q)\times\frac{\partial l_s}{\partial p_{sm}}\frac{\partial\partial V_t^k}{\partial l_s}\mathcal{N}_t^k\mathrm{d}q+2\alpha_{sm}\left(I_2-\frac{\partial l_s}{\partial p_{sm}}\right)(p_{sm}-l_s). \quad (38)$$

Solving this optimization problem follows the same lines as those in the previous section except that there is now an extra term representing the formation of the agents. Employing (16) and $\gamma_{sm}$, we have

$$\frac{\partial\mathcal{G}}{\partial p_{sm}}=-2M_{V_{sm}}(C_{V_{sm}}-p_{sm})+\gamma_{sm}+2\alpha_{sm}\left(I_2-\left(M_{V_{sm}}I_2+\sum_{p_{tr}\in\{p_{sm},\mathcal{N}_{p_{sm}}\}}M_{\partial V_{tr}}p_{tr}^{\top}\right)\times\left(M_{V_s}I_2+M_{\partial V_s}l_s^{\top}-\sum_{b=1,k\in\mathcal{K}'}^{n_{bs}}M_{\partial V_{sb}}^{k}p_{sb}^{\top}\right)^{-1}\right)(p_{sm}-l_s). \quad (39)$$

The gradient descent method results in the following expression for the agents dynamics assigned to maintain a certain formation through the given formation factor

$$\dot{p}_{sm}=u_{sm}=K_{sm}\left(-\frac{\partial\mathcal{G}}{\partial p_{sm}}\right)=K_{sm}\left(2M_{V_{sm}}(C_{V_{sm}}-p_{sm})-\gamma_{sm}-2\alpha_{sm}\Lambda_{sm}(p_{sm}-l_s)\right), \quad (40)$$

where $\Lambda_{sm}$ is the multiplicative formation term as in (39).

**Remark 4.** The convergence of the agents position to their local optima by using the proposed formation control law in (40) and $\Lambda_{sm}$ is ensured by considering the Lyapunov function as $V=\mathcal{G}$. The proof follows the same lines as those in Theorem 2.2.

Remark 4 implies that the agents converge to their associated local minima while maintaining a desired formation. In fact, changing the formation factor can change the relative distance of the

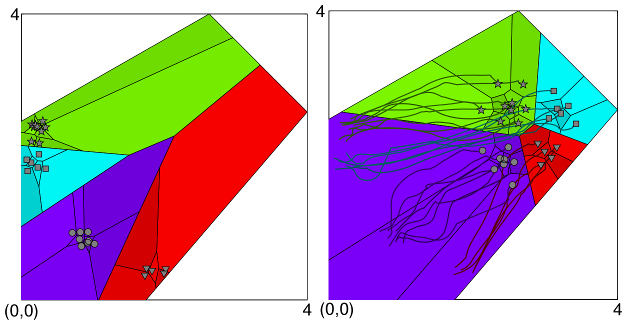


**Fig. 2.** Initial (left) and final configurations (right) for four teams of 5 (asterisk), 9 (star), 6 (square) and 8 (circle) agents, where similar markers represent the agents belonging to the same team.

agents from their respective nuclei resulting in a different formation. As an extension of the present work and in a more generic scenario, the time-varying formation factor might be considered to represent any change in the formation due to, e.g., the changing environment or the assigned tasks.

## 4. Simulation results and discussion

To examine the capabilities of the proposed approaches, the team-based partitioning is validated using numerical examples. As discussed in the previous section, the structure of the teams and agents is assumed to be known *a priori*. The proposed team-based partitioning is employed to deploy four teams consisting of 5, 6, 8 and 9 agents. The designed algorithm deploys the teams of agents in the main region while optimizing the underlying deployment problem within each team. The importance function is considered to be

$$\varphi(x, y) = \exp\left(-\frac{(x-3)^2 + (y-2.5)^2}{2\sigma^2}\right), \tag{41}$$

where the variance is $\sigma^2 = 0.3$. In this example, the agents take different formations within the Voronoi of their associated team to fulfill the desired objective(s) such as a required communication radius to ensure a consistent communication for agents inside each team or any other formation-related task. This can be viewed as a potential representation of heterogeneity when deploying a number of agents that pursue different tasks because of their coverage capabilities. The configuration shown in Fig. 2 is achieved by choosing different formation coefficients $\alpha_{tm}$ for each agent within each team. The gradient descent-based control gain is chosen to be $K_{sm} = 0.01$. The formation factor assigned to each agent combined with the coverage term allows for placing agents in different relative distances from their nucleus. Table 1 shows the formation coefficients of each agent belonging to different teams. The agents tend to converge to a certain formation due to the relative impact of the associated coverage and formation terms. This relative impact is controlled by choosing a relatively lower or higher formation coefficient to enforce the agents to stay closer to or further away from the nucleus of their team. The higher formation term implies a stronger tendency to bring the agents towards the nucleus of the team by increasing the formation coefficient while the coverage term attempts to scatter them over the teams' associated Voronoi. Finally, the proposed team framework requires the computation of a lower number of edges. This is due to the fact that edges shared among agents that belong to different teams in their neighborhood are the same as the edges of their respective team Voronoi cells. Therefore, agents belonging to the boundary groups will require less computational power in order to maintain their respective Voronoi cells.

**Table 1**
Formation coefficients for the four teams of agents shown in Fig. 2.

| | | | | | | | | |
|---|---|---|---|---|---|---|---|---|
| $\alpha_{11}$ | $\alpha_{12}$ | $\alpha_{13}$ | $\alpha_{14}$ | $\alpha_{15}$ | | | | |
| 0.6 | 0.6 | 0.9 | 4 | 0.6 | | | | |
| $\alpha_{31}$ | $\alpha_{32}$ | $\alpha_{33}$ | $\alpha_{34}$ | $\alpha_{35}$ | $\alpha_{36}$ | | | |
| 0.8 | 3 | 3 | 0.9 | 5 | 0.8 | | | |
| $\alpha_{41}$ | $\alpha_{42}$ | $\alpha_{43}$ | $\alpha_{44}$ | $\alpha_{45}$ | $\alpha_{46}$ | $\alpha_{47}$ | $\alpha_{48}$ | |
| 0.3 | 0.3 | 1 | 0.9 | 1 | 0.3 | 0.5 | 0.9 | |
| $\alpha_{21}$ | $\alpha_{22}$ | $\alpha_{23}$ | $\alpha_{24}$ | $\alpha_{25}$ | $\alpha_{26}$ | $\alpha_{27}$ | $\alpha_{28}$ | $\alpha_{29}$ |
| 0.7 | 0.7 | 0.2 | 0.7 | 0.2 | 0.2 | 0.2 | 0.2 | 0.2 |

## 5. Concluding remarks

To adapt to the complexity of the coverage problems in real world applications, a team-based coverage approach is presented in this paper. The proposed approach solves the problem in a more local way leading to the partitioning of the main region into multiple subregions associated with multiple deployed teams. This is beneficial to handle tasks that require a more diverse group of robots due to their complexity. Due to the diversity of the coverage problems, the need for deploying robots with different dynamics or communication characteristics might be inevitable, and hence, the proposed team-based approach is an attempt to modify and improve existing methods in a way that the teams of robots can divide the region among themselves based on their capabilities. This provides the means for autonomous deployment of multiple teams of robots when there is a need for different types of robots from both communication and dynamics perspectives.

## References


Abbasi, F., Mesbahi, A., & Velni, J.M. (2016). Team-based coverage control of moving sensor networks. In *American control conference* (pp. 5691–5696).

Atınç, G. M., Stipanović, D. M., & Voulgaris, P. G. (2014). Supervised coverage control of multi-agent systems. *Automatica*, *50*(11), 2936–2942.

Cortes, J., Martinez, S., Karatas, T., & Bullo, F. (2004). Coverage control for mobile sensing networks. *IEEE Transactions on Robotics and Automation*, *20*(2), 243–255.

Kantaros, Y., Thanou, M., & Tzes, A. (2015). Distributed coverage control for concave areas by a heterogeneous Robot–Swarm with visibility sensing constraints. *Automatica*, *53*, 195–207.

Lee, S., Diaz, Y., & Egerstedt, M. (2015). Multirobot control using time-varying density functions. *IEEE Transactions on Robotics*, *31*(2), 489–493.

Nowzari, C., & Cortés, J. (2012). Self-triggered coordination of robotic networks for optimal deployment. *Automatica*, *48*(6), 1077–1087.

Patel, R., Frasca, P., & Bullo, F. (2013). Centroidal area-constrained partitioning for robotic networks. *Journal of Dynamic Systems, Measurement, and Control*, *136*(3), 137–156.

Schwager, M., Rus, D., & Slotine, J. (2009). Decentralized, adaptive coverage control for networked robots. *International Journal of Robotics Research*, *28*(3), 357–375.

Sharifi, F., Chamseddine, A., Mahboubi, H., Zhang, Y., & Aghdam, A. (2015). A distributed deployment strategy for a network of cooperative autonomous vehicles. *IEEE Transactions on Control Systems Technology*, *23*(2), 737–745.

Song, C., Liu, L., Feng, G., & Xu, S. (2016). Coverage control for heterogeneous mobile sensor networks on a circle. *Automatica*, *63*, 349–358.

Stergiopoulos, Y., & Tzes, A. (2013). Spatially distributed area coverage optimisation in mobile robotic networks with arbitrary convex anisotropic patterns. *Automatica*, *49*(1), 232–237.


**Farshid Abbasi** received his B.Sc. and M.Sc. degrees in Mechanical Engineering both from University of Tabriz in 2007 and 2010, respectively. In January 2013, he joined Complex Systems Controls Lab at the University of Georgia, where he received a Ph.D. in dynamic systems and controls in Dec. 2016. His research interests include multi-agent systems, cooperative control, machine learning and system identification methods focusing on complex nonlinear processes. He is currently with ASML as a controls and system identification research scientist where his research focuses on developing new control and data analysis techniques to enhance rapidly changing semiconductor technologies.

**Afshin Mesbahi** received his B.Sc. degree in electrical engineering from Semnan University in 2007. He received his M.Sc. and Ph.D. degrees in electrical engineering from Sharif University of Technology in 2009 and 2014 respectively. Dr. Mesbahi is currently working as a Postdoctoral Research Associate with Rohani Lab at University of Georgia. Previously, he was a Postdoctoral Research Associate with the Complex Systems Control Laboratory at University of Georgia (November 2014–April 2016) and one of the Visiting Research Scholars with New Mexico State University (March 2013–June 2013) and University of L'Aquila in Italy (October 2012–February 2013). His current research interests focus primarily on distributed coordination algorithms, matrix decomposition, power grids, stochastic modeling of epidemiology, and generally lie in the area of modeling, analysis, and control of large-scale networks.

**Javad Mohammadpour Velni** received B.S. and M.S. degrees in electrical engineering from Sharif University of Technology and University of Tehran, Iran, respectively, and Ph.D. degree in mechanical engineering from University of Houston, TX. He joined the University of Georgia as an assistant professor of electrical engineering in Aug. 2012. Prior to that, he was with the University of Michigan, where he worked in the naval architecture & marine engineering dept. from Oct. 2011 to Jul. 2012. He was also a Research Assistant Professor of mechanical engineering at University of Houston from Oct. 2008 to Sep. 2011 and a Research Associate at the same institution from Jan. 2008 to Sep. 2008. He has published over 100 articles in international journals and conference proceedings, served in the editorial boards of ASME and IEEE conferences on control systems and edited two books on control of large-scale systems (published in 2010) and LPV systems modeling, control and applications (published in 2012). His current research interests are in secure control of cyber physical systems (and in particular, smart grids), coverage control of heterogeneous multi-agent systems, and data-driven approaches for model learning and control of complex distributed systems.