2014 American Control Conference (ACC)
June 4-6, 2014. Portland, Oregon, USA

Stabilizing Dynamic Control Design for Systems with Time-Varying Delay in Control Loop

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***Abstract*— Synthesis of n^{th}-order dynamic systems with time-varying delay in the control loop is considered in this paper. First-order Padé approximation is sought to solve the infinite-dimensional problem of the pure delay. Although the approximation describes the problem in a finite-dimensional state space, it poses internal dynamics instability inherited from the resulted non-minimum phase system. The unstable internal dynamics restricts the system closed-loop bandwidth and leads to an imperfect tracking performance. To circumvent this problem, the overall system dynamics is explored in terms of unstable internal dynamics and input/output pairs. The system internal dynamics is used to design a parameter-varying dynamic compensator which stabilizes the internal dynamics based on a desired tracking error profile. The presented dynamic compensator is used to develop a dynamic controller whose parameter-varying gains are explicitly determined in a systematic and straightforward manner. The proposed approach is used to design a controller for a spark ignition lean-burn engine with large time-varying delay in the control loop. The results are demonstrated against a baseline PI controller combined with a parameter-varying *Smith* predictor to compensate for the time-varying delay.**

I. Introduction

Control systems usually operate in the presence of delay which is the time that it takes to acquire the relevant information for decision-making, creating control decision and executing those decisions [1]. A feedback system, which is stable without delay may experience significant performance deterioration or even become destabilized while subject to some delay [2]. Some examples of time delay systems extensively studied in the literature include internal combustion engines [3], electrical motors [4] and inverters [5], robotics [6], tele-surgery [7], unmanned vehicles [8], decentralized and collaborative control of multi-agents [9], haptics [10], adaptive combustion control [11], and chemical processes with transport delays [12].

Stability analysis and control design for time-delay systems has been a subject of great practical and theoretical importance in the control system community for decades. Interested reader is referred to [13], [14], [15] and the numerous references therein. Interest in realizing the effects of delays and designing stabilizing controllers has been divided into two main directions, namely, delay-dependent stability and delay-independent stability criteria. However, the effect of delays becomes more pronounced for delay-dependent analyses which are, typically, less conservative than the delay-independent ones. There have been great efforts devoted to the stability analysis of delayed system in the past decade [16], [17] where Lyapunov-Krasovskii functionals [18] and Lyapunov-Razumikhin functions [19] have been frequently used for stability analysis of time-delay systems. In the present paper, we will use a different delay-dependent strategy compared to those in the literature to synthesize the system dynamics and design the corresponding controller.

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In this paper, we will use the first order Padé approximation to transfer the actual infinite-dimensional time delay system into a finite-dimensional form where the delay is considered as a time-varying parameter. A new dynamic compensator will then be proposed to stabilize the unstable internal dynamics. The presented compensator will enable the closed-loop system to track the desired tracking error dynamics while making it robust against unmatched perturbations. The control input will be then constructed based on the dynamic compensator and provide the actual system with the robustness and zero steady-state tracking error properties. The contribution of the paper are: (i) developing a systematic approach to design a parameter-varying controller for n^{th}-order systems with time-varying delay in the control loop; (ii) examining applicability of the approach for both delay and non-minimum phase systems; (iii) providing a systematic method to obtain explicitly the parameter-varying gains of the controller with a low computational effort.

The outline of the paper is as follows. Section II presents the system dynamics reconfigured into a form appropriate for the compensator and corresponding controller design. Application of the proposed method to the control design problem for spark ignition engines with time-varying delay is described in Section III. Finally, Section IV concludes the paper.

II. The Control Design

Consider an n^{th}-order continuous-time dynamic system with a time-varying delay in the control as

$$\frac{d^n}{dt^n}y(t) + \sum_{i=0}^{n-1} \alpha_i \frac{d^i}{dt^i}y(t) = \beta u(t-\tau), \tag{1}$$

where $\tau > 0$ is a time-varying delay and $y(t), u(t) \in \mathcal{R}$ are the system output and input, respectively. The solution of

(1) is infinite dimensional due to the delay inclusion in the control loop. A possible way to solve the infinite-dimensional problem of the pure delay is the use of Padé approximation. This approximation represents the system (1) in the finite-dimensional form in the expense of introducing unstable internal dynamics into the system dynamics. Such a system, generally referred to as a non-minimum phase system, restricts the application of classical control design techniques and still remains a challenging problem. However, in the present paper, we will use a first order Padé approximation to design a dynamic controller to stabilize the system (1) while making it robust against unmatched perturbation and tracking the reference signal with desired tracking error dynamics. Consider the first order Padé approximation as

$$\tau\dot{u}(t-\tau)+2u(t-\tau)\simeq-\tau\dot{u}(t)+2u(t). \tag{2}$$

By substitution of (2) into (1), it can be rewritten as

$$\frac{d^{n+1}}{dt^{n+1}}\hat{y}(t)+\sum_{i=0}^{n}\hat{\alpha}_i(\tau)\frac{d^i}{dt^i}\hat{y}(t)=\beta_1(\tau)\frac{d}{dt}u(t)+\beta_0(\tau)u(t), \tag{3}$$

where $\hat{\alpha}_i(\tau)$'s, are the corresponding delay-dependent coefficients, $\beta_0(\tau)=2\tau^{-1}$, $\beta_1(\tau)=-1$ and $\hat{y}(t)$ is the approximated output. It is noted that, (3) is a non-minimum phase system whose order has been increased by one due to the utilized Padé approximation. Eq. (3) can be represented in the state-space form as

$$\begin{array}{rcl}\dot{x}_1(t)&=&x_2(t)\\ \vdots&&\\ \dot{x}_{n+1}(t)&=&-\sum_{i=1}^{n}\hat{\alpha}_{i-1}(\tau)x_i(t)+u(t)\\ \hat{y}(t)&=&[\beta_0(\tau),\ \beta_1(\tau),\ 0,\ \ldots,\ 0]x(t).\end{array} \tag{4}$$

Obviously, the relative degree of (4) is now smaller than the system order $n+1$. The relative degree of a linear system is simply the difference between the degree of the denominator and numerator of the transfer function. However, for state-space representation (4), the relative degree *r* can be obtained using Lie notation [23] if

$$\begin{array}{c}(i)L_gL_f^kh(x)=0\\ (ii)L_gL_f^{r-1}h(x)\neq0\\ f=[x_2,\ldots,\sum_{i=1}^{n}\hat{\alpha}_{i-1}(\tau)x_i]^{\mathrm{T}},\ g=[0,\ldots,1]^{\mathrm{T}},h=\hat{y}.\end{array} \tag{5}$$

For the system (4) with order of *n*+1, the relative degree is obtained to be $r=n$. Hence, there will be a first order internal dynamics whose stability influences the stability of the system (4). To explore the internal dynamics we will use the following proposition which represents the system in terms of n^{th}-order input/output pairs and a first-order internal dynamics.

Proposition 1. *Consider the set*

$$\begin{array}{rcl}\phi_1(x)&=&h(x)\\ \phi_2(x)&=&L_fh(x)\\ \vdots&&\\ \phi_n(x)&=&L_f^{n-1}h(x)\end{array} \tag{6}$$

It is always possible to find a function $\phi_{n+1}(x)$ such that the mapping $\Phi(x)=[\phi_1(x),\ldots,\phi_{n+1}(x)]^T$ has nonsingular Jacobian matrix.

Furthermore, it is always possible to find $\phi_{n+1}(x)$ such that $L_g\phi_{n+1}(x)=0$.

Proof. Since the relative degree n is non-zero, the vector g is non-zero and, thus, the distribution $G=\text{span}\{g\}$ is nonsingular. Hence, by Frobenius's Theorem [23], one can deduce the existence of n real-valued functions $\lambda_1(x),\ldots,\lambda_n(x)$ such that

$$\text{span}\{d\lambda_1,\ldots,d\lambda_n\}=G^{\perp}. \tag{7}$$

Suppose $G\cap(\text{span}\{dh,dL_fh,\ldots,dL_f^{n-1}h\})^{\perp}\neq0$ which means that the vector g annihilates all the covectors in $\text{span}\{dh,dL_fh,\ldots,dL_f^{n-1}h\}$. This is in contradiction with $\langle dL_f^{r-1}h,g\rangle$ being non-zero. Hence,

$$\dim(G^{\perp}+\text{span}\{dh,dL_fh,\ldots,dL_f^{n-1}h\})=n+1. \tag{8}$$

By using (7) and (8) and the fact that the row vectors $dh,dL_fh,\ldots,dL_f^{n-1}h$ are linearly independent and dimension of its span is n, it is possible to find function λ_1 where the $n+1$ differentials $dh,dL_fh,\ldots,dL_f^{n-1}h,d\lambda_1$ are linearly independent and

$$\langle\lambda_{n+1}(x),g(x)\rangle=L_g\lambda_{n+1}(x)=0 \tag{9}$$

and this establishes the required result. □

Now, consider the system in the new coordinates $z_i=\phi_i(x),\ 1\leq i\leq n+1$. The new system description for $1\leq i\leq n$ can be expressed as

$$\begin{array}{rcl}\dot{z}_1(t)&=&L_fh(\bar{\Phi})=\phi_2(\bar{\Phi})=z_2(t)\\ \vdots&&\\ \dot{z}_{n-1}(t)&=&L_f^{n-1}h(\bar{\Phi})=\phi_n(\bar{\Phi})=z_n(t)\\ \dot{z}_n(t)&=&L_f^nh(\bar{\Phi})+L_gL_f^{n-1}h(\bar{\Phi})u(t),\end{array} \tag{10}$$

where $\bar{\Phi}=\Phi^{-1}(z(t))$. Moreover, the description of the system in the new coordinates for $n+1$ can be written as

$$\dot{z}_{n+1}(t)=L_f\phi_{n+1}(\bar{\Phi}). \tag{11}$$

Hence, the system dynamics (4) can be expressed in the new coordinates which indeed satisfies the Proposition 1.

$$\begin{array}{rcl}z_i(t)&=&\beta_0(\tau)x_i(t)+\beta_1(\tau)x_{i+1}(t)\ for\ 1\leq i\leq n\\ z_{n+1}(t)&=&x_1(t).\end{array} \tag{12}$$

The overall system dynamics can be then represented as

$$\begin{array}{rcl}\dot{\xi}(t)&=&R(\tau)\xi(t)+S(\tau)\eta(t)+\beta(\tau)u(t)\\ \dot{\eta}(t)&=&P(\tau)\xi(t)+Q(\tau)\eta(t),\end{array} \tag{13}$$

where $R(\tau)$, and $S(\tau)$ are matrices with suitable dimensions, $P(\tau)=-\beta^{-1}$ and $Q(\tau)=2\tau^{-1}$ are time-varying coefficients depending on the delay, $\xi=[z_1\ \ldots\ z_n]^T$ and $\eta=z_{n+1}$. The first equation in (13) is generally referred to as input/output pairs and explicitly relates the input to the output of the system. The second equation in (13) is called the internal dynamics of the system (4) and is influenced implicitly through the first nth order equation of (13). Now, the stability of the internal dynamics should be checked for the control design problem. To do so, the zero dynamics of the system may be sought as an alternative way which in fact is obtained by zeroing the output of the system $\xi(t)$ and its successive derivatives. This can be obtained for the internal dynamics (13) when $\xi(t) = 0$, i.e., $\dot{\eta}(t) = Q(\tau)\eta(t)$. For all the positive delays $\tau > 0$, $Q(\tau) = 2\tau^{-1}$ is positive and hence the zero dynamics of the system is unstable. Consequently, the internal dynamics (13) is characterized as a non-minimum phase system. Achieving the perfect tracking for such systems is difficult since in addition to the system desired performance, the control system should stabilize the internal dynamics as well. Such an excessive control responsibility for non-minimum phase systems restricts the application of many conventional control techniques, which have been primarily developed for minimum phase systems where the controller operates on the tracking error signal [24]. In this paper, we will design a dynamic feedback control system which operates on the compensated error signal instead. The compensated tracking error signal in fact will stabilize the internal dynamics while making it robust against unmatched perturbations. The stability of the proposed controller will be proved and its performance will be validated using an illustrative example for a lean-burn engine with a large time-varying delay. Consider the second equation in (13), i.e.,

$$\dot{\eta}(t) = P\xi(t) + Q\eta(t) + \phi_\eta(t) \tag{14}$$

where $\xi(t)$ is the system output. The bounded unmatched perturbations $\phi_\eta(t)$ is also included in (14) to address a more general scenario. The control objective is to track the reference output profile $\xi^*(t)$ while stabilizing the internal dynamics (14).

Assumption 1. It is assumed that the lth-order time-derivative of unmatched perturbation $\phi_\eta(t)$ is zero, i.e., $\frac{d^l}{dt^l}\phi_\eta(t) = 0$. The same assumption is made for the reference tracking profile $\xi^*(t)$.

To investigate the control design process and stabilize the unstable internal dynamics (14), a dynamic compensator is first proposed in the following theorem.

Theorem 1. *Consider the following dynamic compensator which operates on the tracking error $e(t)$ and the internal dynamics $\eta(t)$*

$$\Big(\frac{d^{k+1}}{dt^{k+1}} + \gamma_k(\tau)\frac{d^k}{dt^k}\Big)\eta(t) = \sum_{i=0}^{k-1} \lambda_i(\tau)\frac{d^i}{dt^i}e(t), \tag{15}$$

where $k = m - 1$ and m is the order of the desired tracking error dynamics

$$\Big(\frac{d^m}{dt^m} + \sum_{i=0}^{m-1} c_i \frac{d^i}{dt^i}\Big)e(t) = 0, \tag{16}$$

where $m \geq l$ and c_i's are chosen based on the desired eigenvalue placement and

$$\begin{aligned} \lambda_i(\tau) &= -P(\tau)\textstyle\sum_{j=0}^{i} Q^{-1-j}(\tau)c_{i-j} \\ \gamma_k(\tau) &= c_k + \textstyle\sum_{j=0}^{k-1} Q^{-1-j}(\tau)c_{k-1-j}. \end{aligned} \tag{17}$$

The dynamic compensator (15) stabilizes the unstable internal dynamics in (14) while tracking the desired reference profile and making it robust against the unmatched perturbation $\phi_\eta(t)$.

Proof. Consider the system internal dynamics (14) rewritten in the following form

$$\dot{\eta}(t) - Q(\tau)\eta(t) = -P(\tau)e(t) + P(\tau)\xi^*(t) + \phi_\eta(t) \tag{18}$$

where $e(t) = \xi^*(t) - \xi(t)$ is the tracking error. By substituting (15) into (18), one can obtain

$$\begin{aligned}\Big[\sum_{p=0}^{k-1}\sum_{i=0}^{1} \bar{Q}_i(\tau)\lambda_p(\tau)\frac{d^{p+i}}{dt^{p+i}} + \frac{d^{k+1}}{dt^{k+1}} + \gamma_k(\tau)\frac{d^k}{dt^k}\Big]e(t) \\ = \Big(\frac{d^{k+1}}{dt^{k+1}} + \gamma_k(\tau)\frac{d^k}{dt^k}\Big)\Pi(t),\end{aligned} \tag{19}$$

where $\Pi(t) = P(\tau)\xi^*(t) + \phi_\eta(t)$, $\bar{Q}_0(\tau) = -Q(\tau)$ and $\bar{Q}_1(\tau) = 1$. Based on Assumption 1, the term in the right hand side of (19) is canceled out and zero steady-state tracking error $e(t)$ is achieved. Rearrangement of the resulted Eq. (19) in descending order of derivatives and replacing the corresponding coefficients from (17) leads to the desired error dynamics (16). □

The presented dynamic compensator (15) will be used to design controller that is able to track the desired tracking profile while stabilizing the internal dynamics of non-minimum phase system (13).

Theorem 2. *Consider the following dynamic control input*

$$\Big(\frac{d^{k+1}}{dt^{k+1}} + \gamma_k(\tau)\frac{d^k}{dt^k}\Big)u(t) = \sum_{p=0}^{k-1}\sum_{i=0}^{n+1} \lambda_p(\tau)\hat{\alpha}_i(\tau)\frac{d^{p+i}}{dt^{p+i}}e(t). \tag{20}$$

The dynamic control (20) which has been built upon the dynamic compensator (15) maintains stability of the system (13).

Proof. Substitution of the control input (20) into the first equation of (13) results in the compensator (15) whose stability analysis was described in Theorem 1. □

To provide stability of the actual delayed system (1) with the proposed controller (20), consider the following proposition.

Proposition 2. *The proposed controller (20) which stabilizes the Padé approximated system (3) can also maintain stability of the system (1) with the delay in the control loop.*

Proof. The loop transfer function of the approximated system can be represented as

$$L_A(s) = \frac{\sum_{p=0}^{k-1}\sum_{i=0}^{1} \bar{Q}_i(\tau)\lambda_p(\tau)s^{p+i}}{s^{k+1} + \gamma_k(\tau)s^k}. \tag{21}$$

Correspondingly, the loop transfer function for the nominal delayed system (1) with the proposed control (20) can be represented as

$$L_N(s) = \frac{\sum_{p=0}^{k-1}\sum_{i=0}^{1} \hat{Q}_i(\tau)\lambda_p(\tau)s^{p+i}}{s^{k+1} + \gamma_k(\tau)s^k}e^{-\tau s} \tag{22}$$

where $\hat{Q}_0(\tau) = -\bar{Q}_0(\tau)$ and $\hat{Q}_1(\tau) = \bar{Q}_1(\tau)$. Obviously, $L_A(s)$ is the Padé approximation of $L_N(s)$ and $|L_A(jw)| = |L_N(jw)|$. Moreover, the designed control input (20) was shown to stabilize the loop transfer function $L_A(s)$ with positive phase margin. Hence, by having $\angle L_A(s) \geq \angle L_N(s)$, the loop transfer function $L_N(s)$ will possess larger positive phase margin compared to the approximated loop transfer function $L_A(s)$. □

III. Application of the Proposed Control on a Lean-Burn Engine with Large Time-Varying Delay

Lean-burn spark ignition engines exhibit significant performance enhancement in terms of tailpipe emissions and fuel economy compared to the common spark ignition engines. They operate at up-stoichiometric air-fuel ratio (AFR) leading to reduced carbon monoxide and hydrocarbons but increased nitrogen oxide (NO_x) levels. The excessive NO_x is stored in the lean NO_x trap (LNT) module which is integrated with the three-way catalyst downstream the universal exhaust gas oxygen (UEGO) sensor. The stored NO_x is released after reaching certain threshold while simultaneous switching of the engine into rich operation converts it to non-polluting nitrogen. Although this process leads to a significant reduction in harmful emissions, it introduces a larger time-varying delay for the gas exiting the cylinder to reach the UEGO sensor. The large time delay restricts the closed-loop system stability and bandwidth. Moreover, wide range of engine operating conditions, the inherent nonlinearities of the combustion process, the large modeling uncertainties and parameter variations pose further challenges to the design of the control system for lean-burn engines.

Consider the engine AFR dynamics as [3], [20]

$$\dot{y}(t) + \tau_s^{-1} y(t) = \tau_s^{-1} u(t - \tau), \tag{23}$$

where $y(t)$ and $u(t)$ are the measured and input AFR, respectively. The parameter τ_s is the time constant of the UEGO sensor and τ is the overall time-varying delay consisting of: (1) cycle delay, τ_c, which is estimated by one engine cycle due to the four strokes of the engine as $\frac{720}{(360/60)N} = \frac{120}{N}$[sec], where N is the engine speed in rpm and (2) gas transport delay which is identified as the time for the exhaust gas to reach the tailpipe UEGO sensor downstream the LNT and can be approximated by $\tau_g = \vartheta / \dot{m}_a$ for an average exhaust temperature, where $\dot{m}_a$ is the air mass flow and ϑ is a constant that should be determined based on the experimental data [20].

Eq. (23) matches Eq. (1) with $n = 1$ and $\alpha_1 = \beta = \tau_s^{-1}$. By using the first order Padé approximation (2), the system dynamics can be represented as

$$\begin{aligned} \dot{x}_1(t) &= x_2(t) \\ \dot{x}_2(t) &= -\hat{\alpha}_0(\tau)x_1(t) - \hat{\alpha}_1(\tau)x_2(t) + u(t) \\ \hat{y}(t) &= \hat{\beta}_0(\tau)x_1(t) + \hat{\beta}_1(\tau)x_2(t), \end{aligned} \tag{24}$$

where $\hat{y}(t)$ is the approximated output and $\hat{\alpha}_0(\tau) = \hat{\beta}_0(\tau) = 2(\tau_s\tau)^{-1}$, $\hat{\alpha}_1(\tau) = (2\tau_s + \tau)(\tau_s\tau)^{-1}$, and $\hat{\beta}_1(\tau) = -\tau_s^{-1}$ are the delay-dependent coefficients. By using definition of (5) one can easily determine that the relative degree of the second order system (24) is $r = 1$. Eq. (12) can be used to explore the system internal dynamics and the input/output pairs by choosing $[z_1(t), z_2(t)]^{\mathrm{T}} = [\beta_0(\tau)x_1(t) + \beta_1(\tau)x_2(t), x_1(t)]^{\mathrm{T}}$. Hence, Eq. (13) can be rewritten as

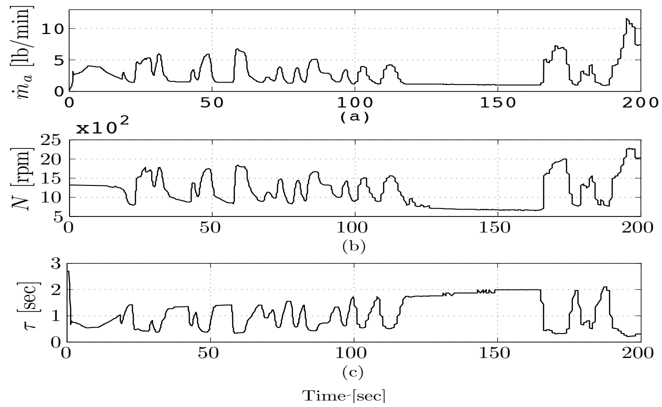

Fig. 1. (a) Air mass flow for the Federal Test Procedure (FTP), (b) Engine speed for FTP, (c) Estimated time-varying delay, $\tau = \tau_c + \tau_g$

$$\begin{aligned} \dot{\xi}(t) &= R(\tau)\xi(t) + S(\tau)\eta(t) + \beta(\tau)u(t) \\ \dot{\eta}(t) &= P(\tau)\xi(t) + Q(\tau)\eta(t), \end{aligned} \tag{25}$$

where $R(\tau) = -(4\tau_s + \tau)(\tau_s\tau)^{-1}$, $S(\tau) = (8\tau_s + 4\tau)(\tau_s\tau)^{-2}$, $P(\tau) = -\tau_s$, $Q(\tau) = 2\tau^{-1}$, and $\beta(\tau) = -\tau_s^{-1}$ are corresponding delay-dependent coefficients. The unstable eigenvalue for the zero dynamics based on $\xi = 0$ is $2\tau^{-1}$, which demonstrates the instability of the internal dynamics of (25) for all time delays.

By considering the desired error dynamics as $\ddot{e}(t) + 1.4\dot{e}(t) + e(t) = 0$ and using Assumption 1 with $k = l = 1$, the proposed dynamic compensator in Theorem 1 can be obtained for the system of interest as

$$\ddot{\eta}(t) + \gamma_1(\tau)\dot{\eta}(t) = \lambda_0(\tau)e(t) \tag{26}$$

where $\lambda_0(\tau) = -0.5\tau_s\tau$ and $\gamma_1(\tau) = 1.4 + 0.5\tau$. Eventually, the corresponding control input can be achieved using Theorem 2 as

$$\begin{aligned} &\ddot{u}(t) + \gamma_1(\tau)\dot{u}(t) \\ &\quad = \lambda_0(\tau)\Big[\ddot{e}(t) + \hat{\alpha}_1(\tau)\dot{e}(t) + \hat{\alpha}_0(\tau)e(t)\Big]. \end{aligned} \tag{27}$$

The experimental data used in this paper are engine air mass flow and engine RPM which have been collected based on a Federal Test Procedure (FTP) as specified in Fig. 1(a) and Fig. 1(b). The engine air mass flow is used to obtain the gas transport delay by $\tau_g = \vartheta/\dot{m}_a$ with $\vartheta = 1.81$ and the engine RPM is used to obtain the cycle delay through $\tau_c = 120/N$. The overall time-varying delay, thus, can be represented in Fig. 1(c) for $0.3\text{sec} \leq \tau \leq 2.7\text{sec}$. It is assumed that the engine is commanded to operate at normalized lean AFR, typically 1.1 or 1.4. The simulations are performed in MATLAB/Simulink® using Runge–Kutta ODE4 for numerical integration.

Figure 2 shows the closed-loop system response for the actual system (solid line) and the approximated non-minimum

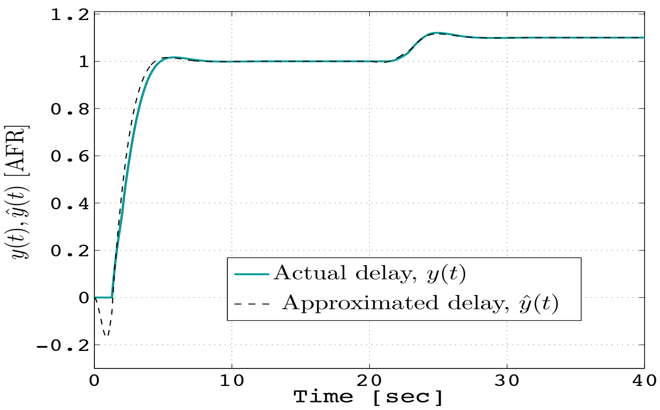

Fig. 2. Closed-loop system performance for the actual system, $y(t)$, and the approximated non-minimum phase system, $\hat{y}(t)$

phase system (dotted line) using the proposed controller. As observed, the controller performs well on the actual system, $y(t)$, and overlaps well with the non-minimum phase system response, $\hat{y}(t)$.

To evaluate the robustness of the proposed controller, a disturbance profile as in Fig. 3 and various delay estimation errors are considered as: (1) nominal delay (dashed line), (2) 20% time delay overestimation (solid line), and (3) 20% time delay underestimation (dotted line). It is shown in Fig. 4 that the closed-loop system is robust against open-loop fuel injector and canister purge disturbance and delay variations. However, for overestimated delays the controller exhibits large oscillations while attenuating the disturbance. This is due to the induced lower bandwidth by the overestimated delay which reduces the system damping and leads to transient oscillations.

The closed-loop system response is compared next with a baseline controller shown in Fig. 5. The baseline controller is a PI controller, $C(s) = K_p(1 + \frac{1}{Ts})$ with parameter T chosen equal to the lag of the system τ_s. The integrator maintains robustness against step disturbances and leads to a first-order set point response with time constant $1/K_p$ [25]. However, very large values of K_p make the closed-loop system unstable due to the delay and the amplification of the measurement noise. Moreover, a parameter-varying Smith predictor, $Z(s) = \frac{1}{\tau_s s+1}(1 - e^{-\tau s})$ is integrated with the PI controller to compensate for large time-varying delays.

Figure 6(a) shows the closed-loop system performance in the presence of the time-varying delay, open-loop disturbance, and the UEGO sensor measurement noise for both proposed controller and the baseline controller. The measurement noise is assumed to be a white noise signal with a power intensity of 10^{-4} which produces a noise with amplitude of 0.05 in the sensor output. It is shown that the proposed controller has attenuated the noise signal more effectively and faster than the baseline controller. It can be seen from Fig. 6(b) that the corresponding control input for the proposed scheme operates with lower noise amplitude compared to the baseline controller. The corre-

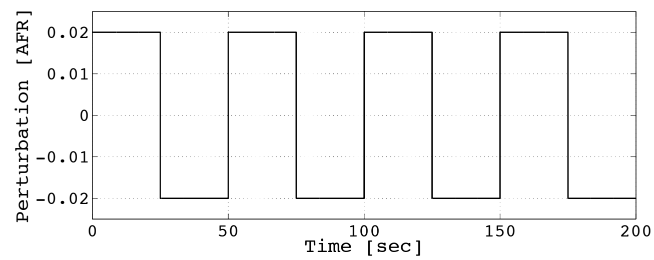

Fig. 3. Typical disturbance profile for the fuel injector and canister purge disturbance

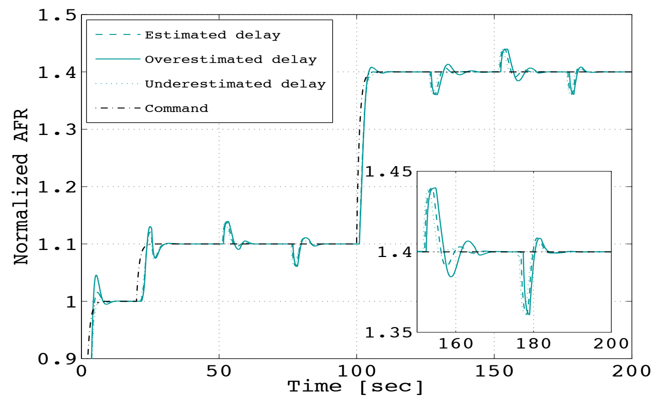

Fig. 4. Closed-loop performance in the presence of the fuel injector and canister purge disturbance and various delay estimation errors

sponding controller parameters have been shown in Fig. 7 where the parameters $\lambda_0(\tau)$ and $\gamma_1(\tau)$ are obtained from Theorem 1 and the other two parameters, $\hat{\alpha}_0(\tau)$ and $\hat{\alpha}_1(\tau)$ are determined from Eq. (4).

IV. Conclusion

A dynamic feedback control design strategy was proposed for n^{th}-order systems with time-varying delay in the control loop. A first order Padé approximation was invoked to tackle the infinite-dimensional problem due to the pure delay. The configured system within finite-dimensional solution has, however, an unstable internal dynamics because of the utilized approximation. A dynamic compensator was then proposed to stabilize the internal dynamics while providing desired tracking error profile. The presented dynamic compensator was then used to design a dynamic controller whose gains were readily calculated based on the desired specifications of the tracking error and dynamic characteristics of

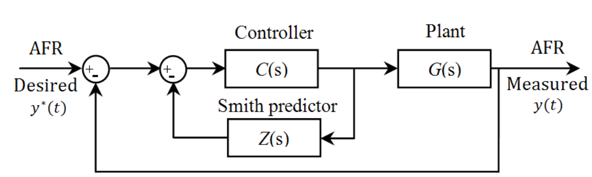

Fig. 5. A baseline PI controller with Smith predictor

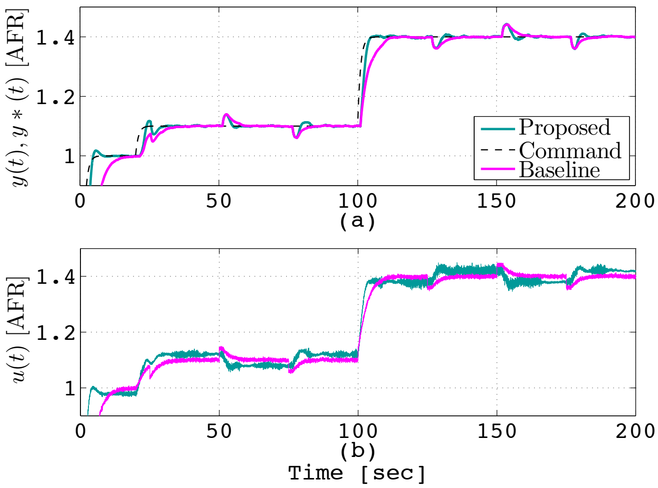

Fig. 6. (a) Output tracking with time-varying delay in the presence of the measurement noise and the fuel injector and canister purge disturbance; (b) The corresponding control input

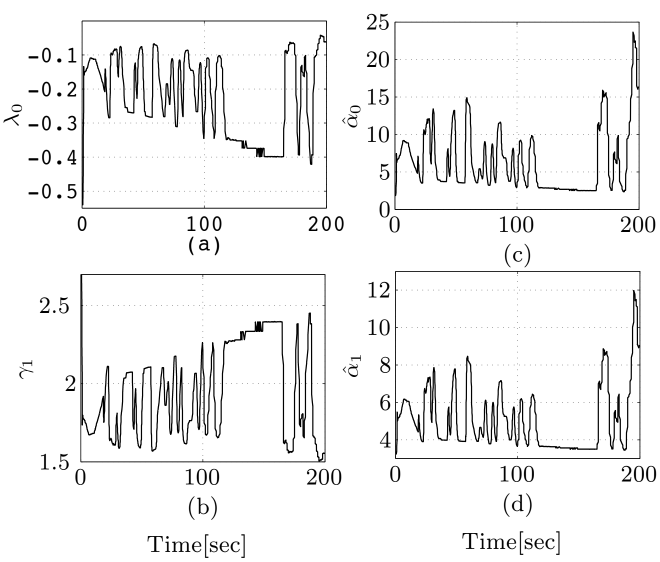

Fig. 7. Delay-dependent time-varying parameters of the controller

the unmatched perturbation. The proposed approach was employed to control a spark ignition engine with a time-varying delay in the control loop. The results obtained for different operating conditions demonstrated the effectiveness of the approach in dealing with disturbances, estimation errors of the delay and noisy output of the system measurements.

V. Acknowledgement

This publication was made possible by NPRP grant 08-398-2-160 from the Qatar National Research Fund (a member of Qatar Foundation).

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