2013 American Control Conference (ACC)
Washington, DC, USA, June 17-19, 2013

Internal Model Control Design for Linear Parameter Varying Systems

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Abstract— **In this paper, we address the design of internal model controllers (IMC) for linear parameter varying (LPV) systems. The design method presented here integrates the IMC and low-pass filter design problems. To this purpose, we formulate the design problem as a linear matrix inequality (LMI) optimization problem by imposing a constraint on the structure of the Lyapunov matrices involved to guarantee robust stability and performance of the closed-loop system. The LMI-based design procedure eventually provides the parameter-dependent state-space matrices corresponding to the IMC controller. Finally, simulation results obtained from a nonlinear system modeled in LPV framework demonstrate the effectiveness of the proposed design method, where we illustrate the improvements achieved by using the LMI-based IMC design over baseline IMC controllers amended with low-pass filters of different bandwidths.**

I. Introduction

Internal model control (IMC) is a well-established and effective design framework for linear systems due to its simple and intuitive controller structure, as well as available design and tuning tools (see [3], [9] and references therein). This method in its original configuration and several modified structures has been successfully applied to various applications from chemical processes to automotive systems (see, *e.g.*, [3], [9] and many references therein). The IMC design approach has been further extended to nonlinear lumped parameter systems showing that the attractive properties of IMC extend beyond the linear control system design and apply to the general nonlinear case [5]. Since 1986 that Economou *et al.* reported in [5] some promising results on application of IMC method for nonlinear systems, there has not been much significant development in this area.

In recent years, there have been some efforts on extending the IMC design method to nonlinear systems in the linear parameter varying (LPV) framework. Xie and Eisaka [16] propose a two degree-of-freedom control design architecture for LPV systems in an IMC framework. This is achieved by introducing coprime factorization and Youla-Kucera parametrization. In Masuda *et al.* [7], the IMC method is extended to the class of max-plus-linear (MPL) systems with LPV structure. It is shown that due to the IMC control law, the proposed method is robust even when the information on the parameter variations is not accurately known. In [14], Toivonen *et al.* extend the IMC design structure for linear systems to that in nonlinear systems, where *velocity-based* linearization is applied to construct a set of linearized models to provide a parameter-varying model of the nonlinear system dynamics. The velocity-form LPV system description captures the relation between the changes in the input and output, while the steady-state information is described by an offset term. To achieve a zero steady-state error when using velocity-based linearized models (due to the offset introduced in the modeling process), a modified IMC structure is then proposed in [14]. Duviella *et al.* utilize an IMC control structure using gain-scheduling technique to deal with a large variation in operating conditions [4]. To this purpose, they rely on an LPV model determined by interpolating several linear time invariant (LTI) models obtained from linearizing the plant around certain operating points.

A more recent approach to IMC design has been proposed by Boulet *et al.* [2] and Yang *et al.* [17], where a linear matrix inequality (LMI)-based formulation of the IMC design problem is sought for. The authors in [2] present an LMI-based approach to robust tunable control, where the tuning strategy is based on the performance robustness bounds of the system and knowledge of the plant uncertainty weighting function. The IMC structure of the controller is adopted together with the additive plant uncertainty, where the design and tuning of the controller employs an approximate solution to the two-disc optimization problem solved over the class of discrete-time finite impulse response (FIR) filters. Yang *et al.* in [17] propose an LMI-based formulation of the IMC design problem for multivariable systems to achieve an acceptable loop performance and small loop couplings. Both of the pioneering work described above are merely applicable to LTI systems.

In the present paper, we propose a method for IMC design in single-input/single-output (SISO) LPV systems. *We note that although the formulation presented in this work is applicable for SISO systems, but it can be extended with no major difficulty to multi-input/multi- output (MIMO) systems as well*. The first step in the proposed method is to derive an LPV model. The model could be obtained from the input-output data by developing a parametrized linear model or from the nonlinear system dynamics by finding an equivalent parameter- dependent state-space representation [15]. The proposed design approach utilizes an LMI-based formulation of the IMC design problem, where the controller state-space matrices are determined by solving an LMI problem. In our design method, the IMC controller can be chosen to have an arbitrary order. In addition, robust stability and performance of the closed-loop system would be guaranteed against the system model uncertainties by appropriately selecting the dynamic weights involved in the

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978-1-4799-0178-4/$31.00 ©2013 AACC

design process. The main contribution of this paper is *the development of an IMC design method for LPV systems, where the controller matrices are determined based on the solution to a convex optimization problem. The derived LMI problem is achieved by imposing a certain structure on some of the underlying inequality variable matrices. One benefit of the design method presented here compared to the method in [4] is that our method eliminates the need for tuning the IMC filter that is crucial in standard IMC design. Besides, the controller order can be selected by the designer and is not bound by the design model.*

The notation used in this paper is standard. $\mathbb{R}$, $\mathbb{R}_+$ and $\mathbb{R}^s$ denote the set of real numbers, the set of non-negative real numbers and the set of real vectors of dimension s, respectively. The matrix I denotes the identity matrix. The notation $(\cdot)^T$ is used to indicate the transpose of a real matrix. In a symmetric matrix, symbol $\star$ in the element (i,j) denotes the transpose of the element (j,i).

II. Problem Statement and Preliminaries

In this section, we first briefly describe the principle and associated properties of the internal model control (IMC) design method. The interested reader is referred to [9] for more details about different approaches to the IMC design for LTI systems. At the end of this section, an introduction to linear parameter varying (LPV) systems will be provided.

A. Internal Model Control Structure and Properties

In the standard IMC design configuration shown in Fig. 1 with the plant and the system model denoted by P and $\tilde{P}$, respectively, the main idea is to include the model of the system, *i.e.*, $\tilde{P}$, into the controller. The IMC structure considers the model $\tilde{P}$ as part of the control loop but not as part of the control design [9]. As described in [12], the plant P is interpreted as the component of the loop that can be controlled, while the plant model $\tilde{P}$ is used for control design purposes. With such an interpretation, some model properties may be designed to intentionally differ from the plant properties, and the model be generally not selected for plant match but for closed-loop robust performance satisfaction [3]. It is noted that for a perfect model, *i.e.*, $\tilde{P}=P$, the control strategy will be simply open loop. We next review general properties of IMC resulting from the structure of the feedback loop shown in Fig. 1. A more detailed discussion and proof of the these properties can be found in [9], [11].

Property 1: (Stability). Assume that the model in IMC structure is exact, *i.e.*, $\tilde{P}(s)=P(s)$. Then, the closed-loop system in Fig. 1 is internally stable if and only if the IMC controller $K(s)$ and the plant $P(s)$ are stable.

Property 2: (Perfect Tracking). Assume that the IMC controller is $K(s)=\tilde{P}^{-1}(s)$ and that the closed-loop system in Fig. 1 is stable. Then, the plant output $y(t)$ tracks the reference input $r(t)$ perfectly, *i.e.*, $y(t)=r(t)$ for $\forall t>0$ and any arbitrary disturbance input $d(t)$.

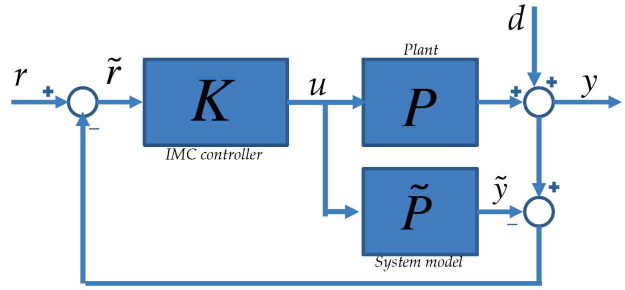

Fig. 1. Standard IMC design configuration for LTI systems

Property 3: (Zero Offset). Assume that the controller steady-state gain is $K(0)=\tilde{P}^{-1}(0)$, and that the closed-loop system shown in Fig. 1 is stable. Then, for any asymptotically constant reference input $\lim_{t\to\infty} r(t)=r_{ss}$ and disturbance $\lim_{t\to\infty} d(t)=d_{ss}$, the steady-state control error is zero.

Property 4: (Robust Stability). Assume that $\tilde{P}(s)$ and $K(s)$ are stable transfer functions, for which the IMC structure of Fig. 1 is stable for an exact model $\tilde{P}(s)=P(s)$. Then, the IMC structure remains stable for multiplicative output uncertainties $\Delta(s)$ if the condition $\|\Delta(s)\tilde{P}(s)K(s)\|_\infty<1$ holds true with $\|.\|_\infty$ denoting the $\mathscr{H}_\infty$ norm.

B. Brief Introduction to LPV Systems

Control of linear parameter-varying (LPV) systems have been studied extensively in the last two decades from stability and performance points of view (see [6], [8], [10] and many references therein). LPV systems provide a systematic way for the gain-scheduling control design of nonlinear systems cast into the LPV forms. A continuous-time LPV system can be described in the following state-space form

$$\begin{aligned} \dot{x}(t) &= A(\rho)x(t)+B(\rho)u(t) \\ y(t) &= C(\rho)x(t)+D(\rho)u(t), \end{aligned} \tag{1}$$

where x and u represent the state vector and the control input vector, respectively; also, y represents the controlled output vector. The LPV parameter vector $\rho(t)=[\rho_1(t),\ldots,\rho_s(t)]$ is assumed to be an arbitrary vector $\rho(t)\in\mathscr{F}^{\nu}_{\mathscr{P}}$, which is not known *a priori* but can be measured or estimated using the existing measurements in real-time. $\mathscr{F}^{\nu}_{\mathscr{P}}$ is the set of allowable parameter trajectories defined as

$$\begin{aligned} \mathscr{F}^{\nu}_{\mathscr{P}} \triangleq\; & \{\rho\in C(\mathbb{R},\mathbb{R}^s):\rho(t)\in\mathscr{P}, |\,\dot{\rho}_i(t)\,|\le\nu_i, \\ & i=1,2,\ldots,s,\ \forall t\in\mathbb{R}_+\}, \end{aligned} \tag{2}$$

where $\mathscr{P}$ is a compact subset of $\mathbb{R}^s$, and $\{\nu_i\}_{i=1}^s$ are non-negative numbers. Next, we present a linear matrix inequality (LMI)-based formulation for the induced $\mathscr{L}_2$-gain of an LPV system using the so-called Bounded Real Lemma (BRL) condition [1].

Lemma 1: The LPV system described by the state-space representation (1) is asymptotically stable and its induced $\mathscr{L}_2$-gain from u to y is less than γ if there exists a continuously differentiable positive definite symmetric matrix Q such *that the following LMI problem holds true*

$$\begin{bmatrix} AQ+QA^T-\dot{Q} & B & QC^T \\ \star & -\gamma I & D^T \\ \star & \star & -\gamma I \end{bmatrix} < 0. \quad (3)$$

III. IMC Design Methods for LPV Systems

In this section, we provide two IMC design methods for LPV systems. The first one is an extension of the standard IMC design from LTI to LPV systems based on the work by Duviella *et al.* [4]. The second method, which is the main contribution of this paper, is an LMI-based formulation to ensure the robust stability and performance.

A. A Simple IMC Design Approach for LPV Systems

In this section, we provide an IMC design approach that is basically an extension from LTI to LPV case by considering the coefficients of the system model to be parametrized [4]. The IMC design configuration is as shown in Fig. 1, with P and $\tilde{P}$ representing the open-loop nonlinear system and nonlinear model capturing the system dynamics, respectively. Then, an LPV representation of the nonlinear system model $\tilde{P}$ can be obtained by either classical Jacobian linearization or describing the system dynamics in a *quasi-LPV* form. *One simple choice* for the IMC law is

$$K_{IMC}(\rho,s) = \mathscr{G}(\rho,s)F(s), \quad (4)$$

where $\mathscr{G}$ is determined based on the inverse of the system transfer function. For more details, the interested reader is referred to [4]. In (4), $F(s)$ is the IMC filter, which can be selected as

$$F(s) = \frac{\zeta}{(1+\lambda s)^k}, \quad (5)$$

with the filter order k chosen to ensure that the IMC controller transfer function K_{IMC} in (4) is proper. It is noted that the selection of the parameter λ leads to a trade-off between stability and transient (tracking) performance. The parameter ζ in the IMC filter is chosen to compensate for the DC gain change caused by pulling out the non-minimum phase part of the system model. In Section IV, we will utilize the inversion-based IMC design procedure described above to examine the performance of the closed-loop system using numerical results.

B. LMI-based IMC Design for LPV Systems

In this section, we propose an IMC design method based on the solution to a linear matrix inequality (LMI) optimization problem. Instead of designing an IMC controller followed by a filter to make the forward path transfer function proper, the proposed design method essentially integrates the design of IMC controller and the filter. This is beneficial since instead of tuning the bandwidth of the filter in (5) to meet the trade-off between robust stability and performance, this can be directly incorporated in the design process.

We first lay out the design problem to be addressed within the loop shaping mixed sensitivity framework. We denote the system's sensitivity transfer function and complementary sensitivity transfer function by S and T, respectively, that are *parameter-dependent operators* for LPV systems due to their dependence on the LPV parameters. We note that S should be small to attenuate the effects of disturbance inputs on the plant and to reduce the tracking error. On the other hand, T should be small to reduce the design sensitivity to measurement errors and reduce control signal. Due to a constraint on the two transfer functions, since $S+T=1$, both S and T cannot be made small as for the linear systems; however, an appropriate design can lead to meeting these conflicting objectives over different frequency ranges. Similar to the standard loop shaping design for LTI systems extended to LPV systems in [1], we seek for the controller $K(s,\rho)$ with the following state-space representation

$$\begin{aligned} \dot{x}_c(t) &= A_c(\rho)x_c(t)+B_c(\rho)\tilde{r}(t) \\ u(t) &= C_c(\rho)x_c(t)+D_c(\rho)\tilde{r}(t), \end{aligned} \quad (6)$$

where the controller state-space matrices A_c, B_c, C_c and D_c are of appropriate dimensions. The controller state-space matrices are obtained from the solution to the following matrix inequality optimization problem

$$\begin{aligned} & \min_{\gamma_S,\gamma_T,K} (\gamma_S+\gamma_T) \\ \text{Subject to:} \quad & \|W_T T\|_{i2} < \gamma_T \\ & \|W_S S\|_{i2} < \gamma_S, \end{aligned} \quad (7)$$

where the subscript $i2$ denotes the induced $\mathscr{L}_2$-gain of the system. In (6), x_c and $\tilde{r}$ represent the controller state vector and the input vector to the controller, respectively. In (7), the dynamic weights W_S and W_T are used to shape the sensitivity and complementary sensitivity transfer functions, respectively. The overall design objective is to ensure that the interconnection of the controller $K(s,\rho)$ in (6) and the plant model $\tilde{P}(s,\rho)$ in (1) in an IMC configuration shown in Fig. 1 is internally stable and the output signal $y(t)$ tracks the reference input $r(t)$. In addition, for a given weighting function W_T corresponding to the multiplicative output uncertainty that represents the error between the plant P and its model $\tilde{P}$, one can also guarantee the robust stability and performance by solving an optimization problem in the form of (7) provided that an extra constraint $\gamma_S+\gamma_T<1$ is added to the set of constraints in (7). This condition is an extension of robust stability and performance analysis conditions for uncertain systems as discussed in detail in [9], [13]

Next, we consider that the dynamic weight on the sensitivity transfer function W_S is described by its state-space matrices (A_S,B_S,C_S,D_S) and the weight on the complementary sensitivity transfer function W_T is described by (A_T,B_T,C_T,D_T). It can be readily shown that for the IMC configuration with a perfect internal model, that is $\tilde{P}=P$, the following relations hold true

$$S = 1-PK, \qquad T = PK,$$

where we have dropped the dependency of the transfer functions on the LPV parameter vector ρ. Based on the relations above, the state-space representation of the transfer functions $W_T T$ and $W_S S$ denoted, respectively, by $(A_{\mathscr{T}}, B_{\mathscr{T}}, C_{\mathscr{T}}, D_{\mathscr{T}})$ and $(A_{\mathscr{S}}, B_{\mathscr{S}}, C_{\mathscr{S}}, D_{\mathscr{S}})$ are given by

$$\begin{aligned} A_{\mathscr{T}} &= \begin{bmatrix} A_T & B_T C & B_T D C_c \\ 0 & A & B C_c \\ 0 & 0 & A_c \end{bmatrix}, \quad B_{\mathscr{T}} = \begin{bmatrix} B_T D D_c \\ B D_c \\ B_c \end{bmatrix} \\ C_{\mathscr{T}} &= \begin{bmatrix} C_T & D_T C & D_T D C_c \end{bmatrix}, \quad D_{\mathscr{T}} = D_T D D_c \end{aligned}$$

and

$$\begin{aligned} A_{\mathscr{S}} &= \begin{bmatrix} A_S & B_S C & B_S D C_c \\ 0 & A & B C_c \\ 0 & 0 & A_c \end{bmatrix}, \quad B_{\mathscr{S}} = \begin{bmatrix} B_S(I - D D_c) \\ -B D_c \\ -B_c \end{bmatrix} \\ C_{\mathscr{S}} &= \begin{bmatrix} C_S & D_S C & D_S D C_c \end{bmatrix}, \quad D_{\mathscr{S}} = D_S(I - D D_c). \end{aligned}$$

Next step is to derive an LMI condition determined from the BRL for each of the two constraints in (7). We first note that if the matrix C_c is fixed, then the two matrices $C_{\mathscr{S}}$ and $C_{\mathscr{T}}$ are independent of the unknown matrices while the matrices $A_{\mathscr{S}}$ and $A_{\mathscr{T}}$ are dependent merely on the unknown controller matrix A_c. Considering two Lyapunov matrices Q_1 and Q_2 corresponding to the two inequality conditions in (3), they are multiplied by $C_{\mathscr{S}}$ and $C_{\mathscr{T}}$, that are fixed, as well as by $A_{\mathscr{S}}$ and $A_{\mathscr{T}}$ that are dependent only on the controller matrix A_c. In addition, the matrices B and D appear affinely in (3) implying that the dependency of the matrix inequality (3) on the controller matrices B_c and D_c is affine. In summary, only the product of the controller matrix A_c and the Lyapunov matrix Q appears in the resulting matrix inequality conditions. To linearize the inequality condition (3), we impose the following structure on the matrix Q

$$Q = diag(Q_a, Q_b), \quad \text{with} \quad Q_b = \lambda_b I \tag{8}$$

where Q_a is determined by solving the LMI problem and λ_b is a tuning parameter.

The simulation results demonstrate that the conditions imposed on the Lyapunov matrix Q in (8) are too restrictive and can lead to an infeasible solution to the LMI problem in (7). For this reason, we propose the following lemma to increase the degrees of freedom by taking advantage of two slack variables V_1 and V_2.

Lemma 2: The LPV system described by the state-space representation (1) is asymptotically stable and its induced $\mathscr{L}_2$-gain is less than γ if there exists a continuously differentiable positive definite symmetric matrix R and matrices V_1 and V_2 such that the following LMI is satisfied

$$\begin{bmatrix} -V_1 - V_1^T & R + V_1 A - V_2^T & V_1 B & 0 \\ \star & V_2 A + A^T V_2^T + \dot{R} & V_2 B & C^T \\ \star & \star & -\gamma I & D^T \\ \star & \star & \star & -\gamma I \end{bmatrix} < 0. \tag{9}$$

Proof: The lemma is a special case of the main result of [18]. The interested reader is referred to [18] for a proof.

As observed, in (9), the matrix R is no longer multiplied by the system matrices; however, the slack variables V_1 and V_2 are now multiplied by the system matrices. Next, we propose a lemma that we will later use in the design of IMC controller.

Lemma 3: The LPV system described by the state-space representation (1) is asymptotically stable and its induced $\mathscr{L}_2$-gain is less than γ if there exists a continuously differentiable positive definite symmetric matrix Y and a symmetric matrix U such that the following LMI condition is satisfied

$$\begin{bmatrix} -2U & Y + (A - I)U & B & 0 \\ \star & AU + UA^T + \dot{Y} & B & UC^T \\ \star & \star & -\gamma I & D^T \\ \star & \star & \star & -\gamma I \end{bmatrix} < 0. \tag{10}$$

Proof: We start with the inequality condition (9), to which we apply the congruence transformation $\mathscr{T} = diag(U, U, I, I)$, with $U = V_1^{-1}$. We finally obtain (10) by imposing an additional constraint on slack variables as $V_1 = V_2 = V_2^T$ to ensure that the derived condition is an LMI problem.

To summarize the discussion in this section, the procedure described below will be used in the design of IMC controller taking advantage of the proposed LMI-based design method.

Procedure to design the IMC controller: Given an LPV representation of a dynamic system in the state-space form (1), the following procedure can be utilized to design an IMC controller to guarantee robust stability and tracking performance.

Step 1: Select the dynamic weights W_S and W_T. The weight W_T can represent the unmodeled dynamic uncertainties (as discussed earlier), and W_S can be chosen to ensure zero tracking error (see Remark 1 below).

Step 2: Select a fixed controller matrix C_c, the controller order, as well as the dependency of the unknown decision matrices including A_c, B_c and D_c with respect to ρ as

$$H = \sum_{j=1}^{p} g_j(\rho) H_j,$$

where H could be any of the LMI matrix variables, $\{g_1, g_2, \ldots, g_p\}$ are the known basis functions, and p is the number of basis functions. Then, solve the LMI optimization problem

$$\min_{\{A_c, B_c, D_c, E, F, \mathscr{X}, \mathscr{Y}, \gamma_S, \gamma_T\}} (\gamma_S + \gamma_T)$$

$$\text{subject to:} \quad \begin{aligned} &\begin{bmatrix} -2F & \mathscr{Y} + (A_{\mathscr{T}} - I)F & B_{\mathscr{T}} & 0 \\ \star & A_{\mathscr{T}} F + F A_{\mathscr{T}}^T + \dot{\mathscr{Y}} & B_{\mathscr{T}} & F C_{\mathscr{T}}^T \\ \star & \star & -\gamma_T I & D_{\mathscr{T}}^T \\ \star & \star & \star & -\gamma_T I \end{bmatrix} < 0 \\ &\begin{bmatrix} -2E & \mathscr{X} + (A_{\mathscr{S}} - I)E & B_{\mathscr{S}} & 0 \\ \star & A_{\mathscr{S}} E + E A_{\mathscr{S}}^T + \dot{\mathscr{X}} & B_{\mathscr{S}} & E C_{\mathscr{S}}^T \\ \star & \star & -\gamma_S I & D_{\mathscr{S}}^T \\ \star & \star & \star & -\gamma_S I \end{bmatrix} < 0 \\ &\mathscr{X} > 0 \\ &\mathscr{Y} > 0, \end{aligned} \tag{11}$$

where the matrices E and F have the following structures

$$E = diag(E_a, E_b), \quad \text{with } E_b = \lambda_S I$$
$$F = diag(F_a, F_b), \quad \text{with } F_b = \lambda_T I,$$

with λ_S and λ_T being two tuning scalars.

Remark 1: It is noted that to ensure zero tracking error, one has to add the condition $T(s=0)=1$ to the set of constraints; however, this additional constraint destroys the convexity of the underlying design optimization problem. Thus, instead of adding this equality constraint, we design a controller to ensure that $S(s=0)$ is very small since $S+T=1$ at any frequency. This can be achieved by choosing the weight W_S to be large at low frequencies.

Remark 2: It is noted that the weights W_S and W_T can be dependent on the LPV parameter vector $\rho(t)$, in which case some of the corresponding state-state matrices become parameter-dependent.

Remark 3: A *sufficient* condition to ensure robust stability and performance is the existence of a solution to the optimization problem in the form of (11) with an additional constraint $\gamma_T + \gamma_S < 1$. It is important to note that this is only a sufficient condition implying that if the optimization problem above does not have a solution, it does not necessarily mean that the robust stability and performance cannot be guaranteed. In fact, the constraints imposed on the structure of the slack variables, as well as the two matrices E and F make the obtained problem only a sufficient condition.

Remark 4: The derivative of the continuously differentiable matrix functions $\mathscr{X}$ and $\mathscr{Y}$ appears in the set of LMIs (11). Considering the dependency of these matrix functions on the LPV parameter vector ρ, the derivative can be substituted by

$$\dot{\mathscr{Z}} = \sum_{i=1}^{s} \frac{\partial \mathscr{Z}}{\partial \rho_i} \dot{\rho}_i, \qquad \mathscr{Z} = \mathscr{X} \text{ or } \mathscr{Y}.$$

Due to the affine dependency of the LMIs with respect to $\dot{\rho}_i$, the LMI conditions in (11) need to be solved only at the vertices, *i.e.*, ν_i and $-\nu_i$.

Remark 5: The derived LMI problem in (11) is infinite dimensional due to its dependence on the LPV parameters. A widely used approach for solving an infinite-dimensional LMI problem is to grid the parameter space [1]. To keep the computational effort feasible, the minimization for a defined grid should be performed, and the constraints for the obtained γ_T and γ_S must be checked on a finer grid. If this check fails, the minimization needs to be repeated for a more dense grid.

Remark 6: An advantage of the proposed LMI-based IMC design method is that the order of the controller can be selected by the designer and is not dictated by the plant model or the weights.

IV. Simulation Results

In this section, we illustrate the results of the two proposed IMC design methods described earlier in this paper.

Example 1: We consider a nonlinear system represented by [10]

$$\begin{aligned} \dot{x} &= -ax + bu \\ y &= \tanh(x) \\ z &= r - y \end{aligned} \tag{12}$$

and assume that the objective is to track an exogenous reference input $r(t)$. In this simple case and for the nominal values of the system parameters $a = b = 1$, there is an obvious equilibrium family that corresponds to zero tracking error. This family can be parameterized by $\rho \in (-1, 1)$ as

$$x_e(\rho) = u_e(\rho) = \tanh^{-1}(\rho), \quad r_e(\rho) = y_e(\rho) = \rho, \quad z_e(\rho) = 0$$

and the deviation variables are

$$\begin{aligned} x_\delta(t) &= x(t) - x_e(\rho), \quad u_\delta(t) = u(t) - u_e(\rho) \\ y_\delta(t) &= y(t) - \rho, \quad r_\delta(t) = r(t) - \rho, \quad z_\delta(t) = z(t). \end{aligned}$$

The corresponding family of linearized plants for the nominal case with $a = b = 1$ is then given by

$$\begin{aligned} \dot{x}_\delta &= -x_\delta + u_\delta, \quad y_\delta = (1-\rho^2)x_\delta \\ z_\delta &= r_\delta - y_\delta. \end{aligned} \tag{13}$$

The LPV representation in (13) is used as the model for the IMC design. Similar to the previous example, we design two IMC controllers and compare the performance of the closed-loop systems.

To solve the LMI problem involved in the design of LMI-based IMC controller, the following considerations are included

- The design weights are selected as $W_S = \frac{s+3}{50(s+0.01)}$ and $W_T = \frac{s+0.5}{20(s+20)}$.
- The controller order is selected to be one.
- Due to the affine dependency of one of the LPV system matrices on ρ^2, we consider the controller state-space matrices, as well as matrices $\mathscr{X}$ and $\mathscr{Y}$ to be affinely dependent on ρ^2.
- We grid the LPV parameter space by choosing 11 equidistant points in the interval $\rho^2 \in [0, 1]$.

To design an IMC controller using the method of [4] described in Section III, we first take an inverse of the transfer function of the linearized system (13) that is dependent on the LPV parameter ρ. Then, due to the structure of the system transfer function, we choose the IMC filter in (5) to be of first order. We performed time simulations using different values of the filter parameter λ. The results using two values of λ is shown next.

To validate the performance of the two controllers using time simulations, we consider the following

- We assume a system model with the same structure as the plant given in (12) with the perturbed parameters $\bar{a} = 2$ and $\bar{b} = 1.1$.
- We introduce a step disturbance with amplitude of 0.1 at $t = 35$ *sec*.
- We restrict the control input to $u \in [-3, 3]$.

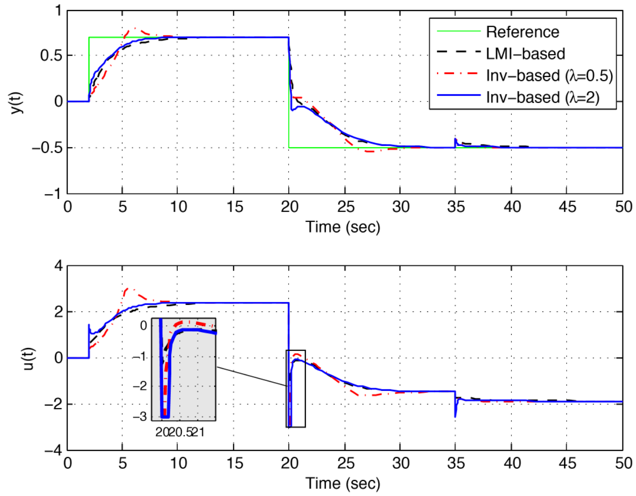

Fig. 2. Closed-loop system output response y and corresponding control input u for Example 1 using the two design approaches for IMC-LPV

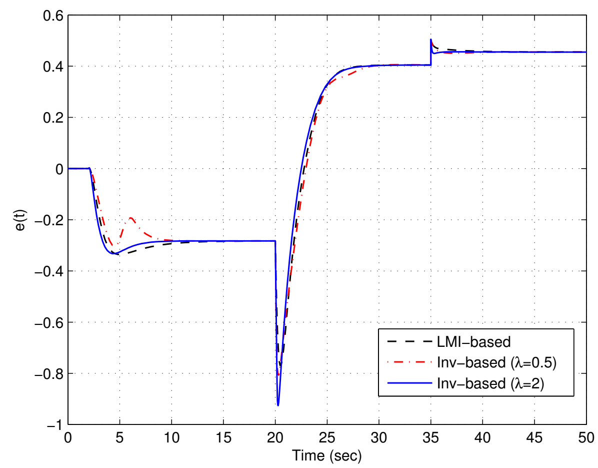

Fig. 3. Error signals corresponding to the three controllers for Example 1

Solving the LMI problem with the tuning parameters $\lambda_T = \lambda_S = 10^{-2}$ results in $\gamma_S = 0.5356$ and $\gamma_T = 0.0046$. Shown in Fig. 2 is the response of the closed-loop system using the proposed LMI-based IMC design procedure, as well as the inversion-based IMC-LPV control described in Section III for two different values of λ, namely, $\lambda = 0.5, 2$. The two subplots in Fig. 2 illustrate the plant output y and control input u corresponding to the two design methods. The plots demonstrate that the LMI-based IMC controller shows an improved transient response compared to the inversion- based controllers for both $\lambda = 0.5$ and $\lambda = 2$. The improvement is obvious in terms of smoother response, no overshoot and quick recovery from the disturbance affecting the system output. We have shown in Fig. 3 the error signal, *i.e.*, the difference between the plant and model outputs, corresponding to the three designs.

V. Concluding Remarks

In this paper, within the standard configuration for the design of internal model controllers, we have presented an LMI-based method to ensure the reference tracking while robust stability and performance of the closed-loop system are guaranteed. The main sources of uncertainties affecting the IMC structure are the measurement errors, as well as inaccuracies of the system model used for the IMC design purposes. To tackle this issue, we have proposed an LMI-based formulation to guarantee the robustness of the design within the IMC design process. It has been shown that the order of the controller is not imposed by the order of the plant or model; instead, it can be selected by the designer and obviously affects the performance of the closed-loop system. Simulation results are shown to demonstrate the viability of the proposed method to ensure robust stability and tracking performance and its advantages compared to the standard inversion-based IMC design extended for parameter-varying systems.

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