

Anti-Windup LPV Control of Pitch Actuators in Wind Turbines

Mona Meisami-Azad, Javad Mohammadpour, Karolos Grigoriadis

Abstract—Operation of wind turbines in the full-load region mandates that the produced power is kept at a rated value to minimize structural loads and thereby reduce fatigue damages. This is usually achieved by pitching the rotor blades in order to limit the aerodynamic torque in high wind speeds. The pitch actuators usually present a hard constraint in terms of the amplitude and rate of saturation. In this paper, we propose a method to address pitch actuator amplitude and rate saturation by designing the anti-windup controllers in the linear parameter varying (LPV) framework. The proposed design method guarantees the closed-loop stability and a prescribed level of performance while it decreases the pitch activity for regulating the generated power to the nominal power during sudden wind gusts. The anti-windup controller designed to minimize the \mathcal{H}_∞ norm of the closed-loop system is gain-scheduled based on the operating condition of the turbine, as well as the states of amplitude and rate saturation of the pitch actuator. The effectiveness of the proposed control design method is demonstrated using the high-fidelity aeroelastic dynamic simulation tool FAST.

I. INTRODUCTION

When the actual plant input is different from the output of the controller as a result of actuator limitations, the controller output will not drive the plant, and the states of the controller will be wrongly updated. Anti-windup control design is an efficient approach employed to account for the actuator saturation nonlinearity that is widely encountered in real-world applications. Desirable design requirements for anti-windup compensation subject to actuator saturation include the stability of the closed-loop system, recovery of the linear design specifications in the absence of saturation (linear performance recovery), and the smooth degradation of the linear performance in the presence of saturation (graceful performance degradation) [11]. There are two schemes for anti-windup control: the first one takes the saturation nonlinearity into account from the beginning of the controller design process, while in the second one a linear controller is first designed followed by a second controller to compensate for the saturation effects. In addition to the actuator amplitude saturation, the rate saturation can also be problematic in some applications such as modern flight control systems. As an example of the first design scheme, Nguyen and Jabbari [12] formulated the anti-windup control design problem for the amplitude and rate saturation nonlinearities in the form of a convex optimization problem through linear matrix inequalities (LMIs). An early review of the anti-windup control methods was reported in [4], in which the anti-windup compensator synthesis problem was addressed in an \mathcal{H}_∞ framework. The authors in [9] collected

some of the recent efforts made on the development of the analysis and synthesis methodologies for systems with actuator saturations.

With the aim of using simple controllers, the generator torque and pitch angle of variable-speed variable-pitch wind turbines are often controlled separately. In low wind speeds, wind turbines are operated at variable speed and fixed pitch in order to capture as much energy as possible. In high wind speeds, the pitch angle is adjusted to regulate the captured power to its rated value whereas the generator torque characteristics often remains unchanged. In fact, the pitch angle actuators generally present a hard constraint on their speed response besides the natural amplitude saturation. To reduce the effects of fatigue loads, the amplitude and rate saturation limits of the pitch actuator should not be reached during normal operation of the turbine. For instance, high frequency load mitigation, as well as power smoothing demands fast and large corrections to the pitch angle that might cause fatigue damage to some mechanical devices. These limitations should be considered in the controller design procedure to avoid the high activity of the pitch actuators, since it could not only damage the pitch actuators but also can give rise to unstable modes of operation [2], [13]. Although actuators are mostly subject to a combination of several constraints, the attention has been mostly paid on position constraints, and the actuator dynamics are usually ignored. Leith and Leithead in [10] described the pitch saturation constraint problem in implementing the control algorithm and compared various methods for anti-windup design related to pitch actuators. The authors also showed that it is almost always the case that only one of the velocity and acceleration constraints proves restrictive in practice and it is sufficient to consider actuators with a single velocity or acceleration constraint. Garelli *et al.* proposed in [5] a compensation method by adding an auxiliary loop and conditioning the reference signal to tackle the amplitude and rate saturation of pitch actuator. The approach in [5] is validated on a small scale wind turbine used for water pumping. In [8], the wind-up problem is addressed for individual pitch control when the controller has an integrator term. The actuator limits are first transformed to multi blade coordinates, and then, by using an auxiliary loop the integrators are implemented so that their states are driven by actual constrained inputs.

In this paper, we address the problem of anti-windup control design for wind turbines with pitch actuator amplitude and rate saturation in the linear parameter varying (LPV) framework. The design method proposed in this paper guarantees the stability of the closed-loop system along with

Mechanical Engineering Department, University of Houston, TX 77004.
Corresponding author's email address: mmeisami@mail.uh.edu

an optimal \mathcal{H}_∞ norm performance while decreasing the pitch actuator activity. The design objective is to regulate the generated power at the nominal power especially in the presence of sudden wind gusts. The \mathcal{H}_∞ anti-windup controller is gain-scheduled based on the operating condition of the turbine, as well as the states of amplitude and rate saturation of the pitch actuator. The effectiveness of the proposed anti-windup control design method in LPV framework is shown using the FAST (Fatigue, Aerodynamics, Structures, and Turbulence) code which is a detailed aeroelastic wind turbine dynamic simulation tool developed by the DOE National Renewable Energy Laboratory (NREL).

The paper is organized as follows. Section II describes the nonlinear model representing the dynamics of the wind turbine and its transformation to an LPV form. Section III formulates the LPV anti-windup control synthesis problem to tackle the actuator saturation. In Section IV, we present the simulation results of applying the proposed method to the wind turbine model in FAST. Section V concludes the paper.

II. WIND TURBINE MODEL

In this section, we describe the development of an LPV model of a three-bladed horizontal axis wind turbine considering the aerodynamic, drive-train, and pitch actuator components. We further show the steps involved in developing an LPV model of the wind turbines considering the turbine operating condition variability and pitch actuator saturation status.

A. Nonlinear Model of Wind Turbines

For the sake of completeness, we briefly review the lumped-parameter models corresponding to different subsystems in a wind turbine.

1) *Aerodynamic Model:* The rotor is considered as a uniform wind extractor transforming kinetic energy from the wind passing through the rotor plane to the mechanical energy at the shaft and through the generator to the electrical power. The torque and power obtained from the rotor can be determined from:

$$Q_r = \frac{1}{2} \pi R^3 \rho v^2 C_Q(\lambda, \beta)$$

$$P_r = \frac{1}{2} \pi R^2 \rho v^3 C_P(\lambda, \beta)$$

where R is the blade length, v is the average wind speed at the turbine hub, ρ is the air density, C_P and C_Q are the power and torque coefficients that are dependent on the tip-speed ratio λ and the collective blade pitch angle β . Tip-speed ratio is defined as $\lambda = \frac{R\omega_r}{v}$ where ω_r is the rotor speed. The turbine used for the simulation and validation purposes in this paper is the NREL 5MW baseline wind turbine [6]. The power coefficient corresponding to this turbine is shown in Figure 1.

2) *Drive-Train Model:* As illustrated in Figure 2, the dynamics of the drive-train system can be represented using the following components: rotor, low-speed shaft, gearbox, high-speed shaft and generator. The inertia of the rotor and the

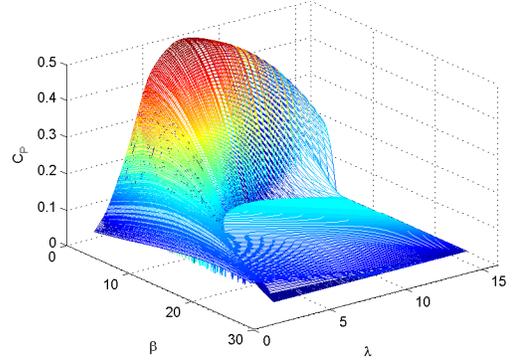


Fig. 1. Power coefficient C_p as a function of tip-speed ratio and pitch angle

total inertia of the gearbox, high-speed shaft, and generator are denoted by J_r and J_g , respectively. A massless viscously damped rotational spring with the damping coefficient B_s and the spring coefficient K_s is assumed to model the low-speed shaft. The equations of motion are described by

$$\dot{\theta}_s = \omega_r - \frac{\omega_g}{N_g}$$

$$J_r \dot{\omega}_r = Q_r - K_s \theta_s - B_s (\omega_r - \frac{\omega_g}{N_g})$$

$$N_g J_g \dot{\omega}_g = -Q_g N_g + K_s \theta_s + B_s (\omega_r - \frac{\omega_g}{N_g})$$

where N_g is the gear ratio, θ_s is the shaft torsion on the rotor side, Q_r is the low-speed shaft torque and Q_g is the generator torque. Transforming the above differential equations into the state-space form leads to

$$\begin{bmatrix} \dot{\theta}_s \\ \dot{\omega}_r \\ \dot{\omega}_g \end{bmatrix} = \begin{bmatrix} 0 & 1 & -\frac{1}{N_g} \\ -\frac{K_s}{J_r} & -\frac{B_s}{J_r} & \frac{B_s}{J_r N_g} \\ \frac{K_s}{J_g N_g} & \frac{B_s}{J_g N_g} & -\frac{B_s}{J_g N_g^2} \end{bmatrix} \begin{bmatrix} \theta_s \\ \omega_r \\ \omega_g \end{bmatrix} + \begin{bmatrix} 0 & 0 \\ \frac{1}{J_r} & 0 \\ 0 & -\frac{1}{J_g} \end{bmatrix} \begin{bmatrix} Q_r \\ Q_g \end{bmatrix} \quad (1)$$

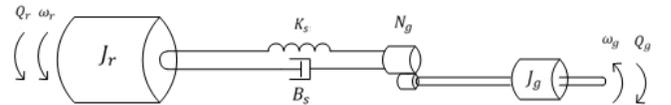


Fig. 2. Drive-train schematic

3) *Pitch Actuator Model:* The pitch actuator is a hydraulic or electromechanical device that allows rotating the turbine blades around their longitudinal axis. The implementation of reliable control strategies is possible mostly through pitch control in commercial wind turbines. In the present paper, we model the pitch actuator as a first-order dynamic system with amplitude and rate saturation as shown in Figure 3, where β and β_d are the actual and desired pitch angles. For the purpose of the present study, we have assumed that β can change from 0 to 30 degrees and varies at a maximum rate of $\pm 2^\circ/s$. It is noted that these saturation limits vary depending on the type of the actuating device, as well as

the turbine power. The amplitude and rate saturation limits are later denoted by u_{lim} and \dot{u}_{lim} in general. In the linear region, the dynamics of the actuator is represented by:

$$\dot{\beta} = -\frac{1}{\tau}\beta + \frac{1}{\tau}\beta_d$$

where τ is the time constant of the actuator. It is noted that the actuator model described above can be extended to multiple-actuator case which can be used in individual pitch control.

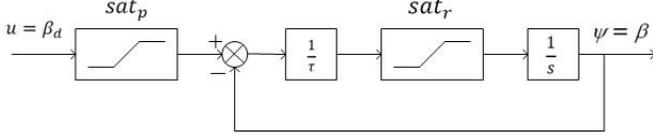


Fig. 3. Actuator model

B. LPV Model of the Wind Turbine

The framework of linear parameter varying systems concerns linear dynamical systems whose state-space representations depend on exogenous non-stationary parameters, as in

$$\begin{aligned} \dot{x} &= A(\theta)x + B(\theta)u \\ y &= C(\theta)x + D(\theta)u \end{aligned} \quad (2)$$

where u is the input, y is the output, and $\theta = [\theta_1, \dots, \theta_s]$ is a vector including exogenous parameters that can be time varying. The first assumption is that the parameters are bounded in both magnitudes and rates of variation as:

$$-\mu_i \leq \theta_i(t) \leq \mu_i, \quad -\rho_i \leq \dot{\theta}_i(t) \leq \rho_i, \quad i = 1, \dots, s. \quad (3)$$

Let Θ denote some specified family of parameter trajectories so that $\theta(\cdot) \in \Theta$. Another important assumption is that the time variation of the LPV parameters is not known *a priori*, but the parameters are assumed to be measurable in real-time. The LPV parameters (also referred to as *the scheduling parameters*) would be further used for adapting the LPV controller matrices.

The overall model of a wind turbine is highly nonlinear due to the nonlinear aerodynamic properties. To develop an LPV model, we first linearize the aerodynamic torque model around an operating envelop characterized by $\bar{\zeta} = (\bar{\omega}_r, \bar{\beta}, \bar{v})$ considering:

$$Q_{r\beta} = \frac{\partial Q_r}{\partial \beta} \Big|_{\bar{\zeta}}, \quad Q_{rv} = \frac{\partial Q_r}{\partial v} \Big|_{\bar{\zeta}}, \quad Q_{r\omega_r} = \frac{\partial Q_r}{\partial \omega_r} \Big|_{\bar{\zeta}}. \quad (4)$$

The derivatives in (4) are now the scheduling parameters of the LPV model. It is noted that in the full-load region of operation, partial derivatives of the aerodynamic torque can be approximated by an affine function with the wind speed as the independent variable, as illustrated in Figure 4. Therefore, here we consider the wind speed to be the only scheduling parameter. The cut-in, rated and cut-out wind speeds for NREL 5MW turbine are 3, 11.4 and 25 m/s, respectively.

Next, we formulate the pitch actuator dynamics with amplitude and rate saturation by defining two new scheduling parameters as the *states of saturation*. We define

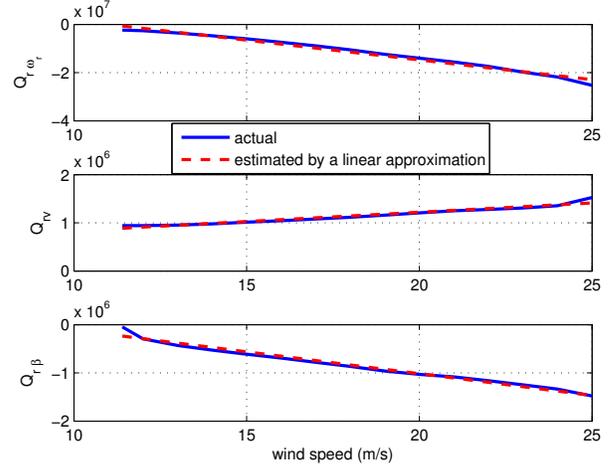


Fig. 4. Partial derivatives of the aerodynamic torque and their linear approximations

$$p = \frac{sat_p(u)}{u}, \quad p = 1 \text{ if } u = 0 \quad (5)$$

$$r = \frac{sat_r(\tau^{-1}(sat_p(u) - \psi))}{\tau^{-1}(sat_p(u) - \psi)}, \quad r = 1 \text{ if } sat_p(u) - \psi = 0 \quad (6)$$

where p and r are equal to one when pitch amplitude and rate are not saturated. Using the above definitions, the dynamics of the actuator shown in Figure 3 can now be represented by

$$\dot{\beta} = -\frac{r}{\tau}\beta + \frac{rp}{\tau}\beta_d \quad (7)$$

where the constraints are $|\beta(t)| \leq u_{lim}$ and $|\dot{\beta}(t)| \leq \dot{u}_{lim}$ in order for the actuator to work in its linear operating region. It can be further proved that if $\dot{u}_{lim} \geq \frac{2}{\tau}u_{lim}$ the rate limiter never saturates [12]. Hence, we assume that the condition $\dot{u}_{lim} < \frac{2}{\tau}u_{lim}$ holds true.

Remark 1: It should be mentioned that the order of integrator and saturation blocks in Figure 3 is important and changes the actuator model considering that different configurations are not equivalent. However, the results can be easily extended to other actuator structures or even actuator dynamics with higher order models.

Finally, the LPV model representing the wind turbine dynamics can be obtained by augmenting the aerodynamic, drive-train, and the pitch actuator subsystem models described above in state-space as follows:

$$\begin{aligned} \begin{bmatrix} \dot{\tilde{\theta}}_s \\ \dot{\tilde{\omega}}_r \\ \dot{\tilde{\omega}}_g \\ \dot{\tilde{\beta}} \end{bmatrix} &= \begin{bmatrix} 0 & 1 & -\frac{1}{N_g} & 0 \\ -\frac{K_s}{J_r} & -\frac{B_s}{J_r} + \frac{Q_{r\omega_r}}{J_r} & \frac{B_s}{J_r N_g} & \frac{Q_{r\beta}}{J_r} \\ \frac{K_s}{J_g N_g} & \frac{B_s}{J_g N_g} & -\frac{B_s}{J_g N_g^2} & 0 \\ 0 & 0 & 0 & -\frac{r}{\tau} \end{bmatrix} \begin{bmatrix} \tilde{\theta}_s \\ \tilde{\omega}_r \\ \tilde{\omega}_g \\ \tilde{\beta} \end{bmatrix} \\ &+ \begin{bmatrix} 0 & 0 \\ \frac{Q_{rv}}{J_r} & 0 \\ 0 & 0 \\ 0 & \frac{rp}{\tau} \end{bmatrix} \begin{bmatrix} \tilde{v} \\ \tilde{\beta}_d \end{bmatrix} \end{aligned} \quad (8)$$

where a variable with $(\tilde{\cdot})$ indicates the deviation of the corresponding variable from its equilibrium point. Comparing

(8) with (2), it is observed that the matrices A and B are functions of the LPV parameters $\theta = [\bar{v} \ r \ rp]^T$ as

$$\begin{aligned} A &= A_0 + Q_{r\omega_r}(\theta_1)A_1 + Q_{r\beta}(\theta_1)A_2 + \theta_2 A_3 \\ B &= Q_{rv}(\theta_1)\bar{B}_1 + \theta_3\bar{B}_2 \end{aligned} \quad (9)$$

where $Q_{r\omega_r}$, $Q_{r\beta}$, and Q_{rv} are defined in (4), and A_0 , A_1 , A_2 , A_3 , \bar{B}_1 and \bar{B}_2 are constant matrices depending only on the turbine parameters.

Similar to the discussion at the end of Section II-A.3, the formulation above can be generalized to allow different levels of saturation limits (both magnitude and rate) for individual pitch actuators.

Remark 2: It is noted that we use real-time information of p and r in the controller adaptation. This is possible because by knowing the controller output u , the plant input ψ , and the limits of amplitude and rate saturation, the two parameters p and r can be easily calculated from (5) and (6), respectively.

III. ANTI-WINDUP LPV CONTROL DESIGN PROCEDURE

In the full-load region of operation of a wind turbine, where the wind speed varies between rated and the cut-out speed, the pitch angle is adjusted to regulate the generator speed at its rated value ω_{gN} in order to maintain the captured power around its rated value P_N . The block diagram of the single-input single-output loop for the pitch control is shown in Figure 5.

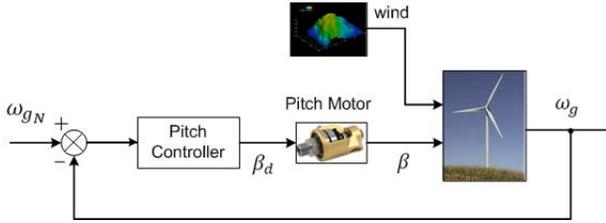


Fig. 5. Wind turbine pitch control block diagram

The proposed anti-windup control design method of the present paper takes advantage of the LPV \mathcal{H}_∞ output feedback control synthesis formulation in [1] using the set of previously defined scheduling parameter vector θ , where not only the operating condition of the turbine is taken into account, but also the amplitude and rate saturation states, *i.e.*, p and r , are included in the design procedure. Figure 6 illustrates the configuration of the augmented plant in (8) with the weights for the synthesis of a mixed sensitivity LPV controller. Dynamic weights are selected to shape the closed-loop transfer functions. The controlled output vector z consists of the weighted input and tracking error, *i.e.*, $z = [z_e, z_u]^T$. The control design objective is to track the nominal generator speed and avoid the high activity pitch especially in the presence of wind gusts. The following weights are selected based on a trial and error procedure:

$$W_e(s) = \frac{0.5(s+5)}{s+0.002}, \quad W_u(s) = \frac{50(0.15s+1)}{0.01s+1} \quad (10)$$

where the weight on the tracking error $W_e(s)$ is a low-pass filter to emphasize the low frequency content of the

error signal. On the other hand, weights on the control input $W_u(s)$ is used to penalize the fast pitch angles at higher frequencies. It should be noted that there is a trade-off between fast pitch correction and the reference tracking performance objectives and this is one of the reasons why the anti-windup control is used to prevent damage to the pitch actuators hardware and provide a reasonable tracking performance. Another method of avoiding high activity pitch actuators proposed in [3] is based on combining the pitch angle control with variable speed operation, where both pitch angle and generator torque are controlled simultaneously. This can be done through decentralizing two individual feedback loops or a multivariable control design.

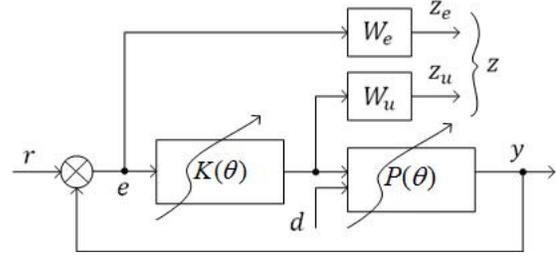


Fig. 6. Configuration of the closed-loop system design based on loop shaping (arrows illustrate the adaption of the blocks with respect to the LPV parameters)

Next, we describe the general anti-windup control design procedure in the LPV framework. First, we consider that the augmented system of the LPV model and the dynamic weights is represented by

$$\begin{aligned} \dot{x} &= A(\theta)x + B_1(\theta)d + B_2(\theta)u \\ z &= C_1(\theta)x + D_{11}(\theta)d + D_{12}(\theta)u \\ y &= C_2(\theta)x + D_{21}(\theta)d. \end{aligned} \quad (11)$$

The gain-scheduled output feedback controller is described in state-space form by:

$$\begin{aligned} \dot{x}_k &= A_k(\theta, \dot{\theta})x_k + B_k(\theta, \dot{\theta})y \\ u &= C_k(\theta, \dot{\theta})x_k + D_k(\theta, \dot{\theta})y \end{aligned} \quad (12)$$

The LPV γ -performance design problem is to find a parameter-dependent controller represented by (12) such that the closed-loop system formed by the interconnection of the open-loop system (11) and the controller (12) is stable and the \mathcal{H}_∞ norm from d to z is less than γ . The following lemma provides necessary and sufficient conditions for solvability and synthesis of the anti-windup LPV control design problem.

Lemma 1: Given the open-loop system in (11) and a scalar $\gamma > 0$, there exists a parameter-dependent controller as in (12) that guarantees closed-loop system stability and \mathcal{H}_∞ norm of less than γ if and only if there exist parameter-dependent symmetric matrices X and Y and the parameter-dependent matrices \hat{A}_k , \hat{B}_k , \hat{C}_k , D_k such that the following

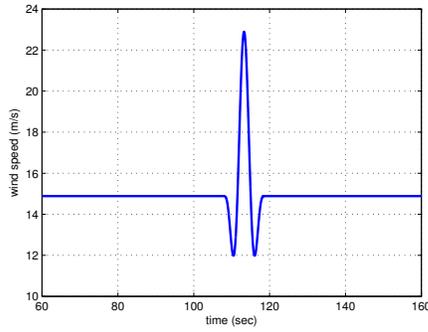


Fig. 7. Wind speed profile for the second set of simulations

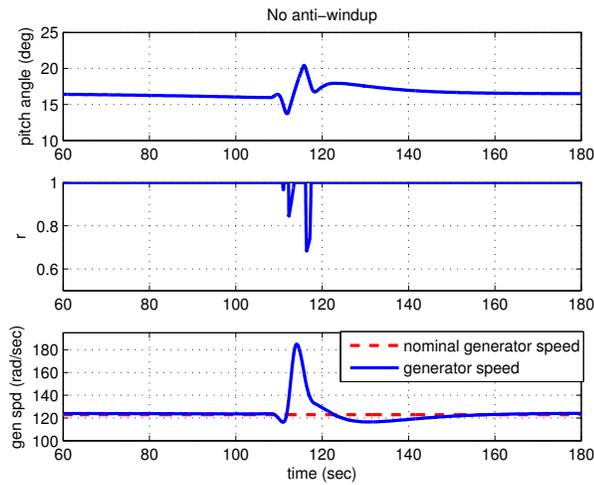


Fig. 8. LPV control design scheduled only on wind speed. Top plot: collective pitch angle; middle plot: parameter r corresponding to the status of actuator rate saturation; bottom plot: nominal and measured generator speed

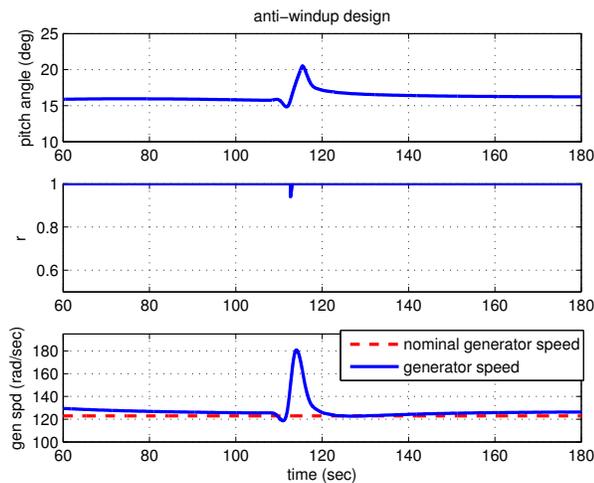


Fig. 9. LPV anti-windup control design gain-scheduled on both wind speed and the parameter r . Top plot: collective pitch angle; middle plot: parameter r corresponding to the status of actuator rate saturation; bottom plot: nominal and measured generator speed

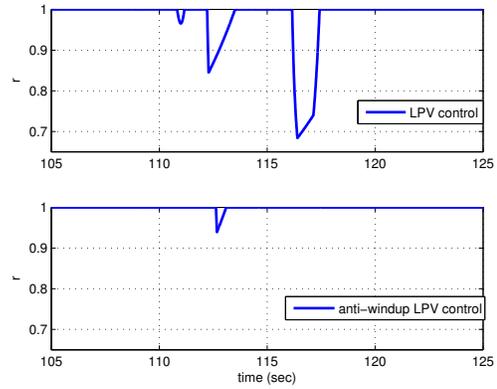


Fig. 10. Zoomed view of the parameter r for the two cases (i) and (ii)

then transformed into an LPV model considering the turbine dynamic variability due to the wind speed variations and actuator saturation status. The simulation results performed in FAST exhibited that the proposed scheme is useful to avoid the actuator saturation and damaging of the pitch motor hardware, while providing a similar tracking performance as the non-anti windup LPV control design case.

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